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THESIS

FORECASTING MARINE CORPS ENLISTED MANPOWER INVENTORY LEVELS WITH UNIVARIATE TIME SERIES MODELS

by

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FORECASTING MARINE CORPS ENLISTED MANPOWER INVENTORY LEVELS WITH UNIVARIATE TIME SERIES MODELS

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ABSTRACT

Accurately forecasting future personnel inventory levels by rank and occupational specialty is a fundamental prerequisite for development of an effective and functional staffing plan. This thesis develops and evaluates univariate time series models to create six- and twelve-month forecasts of Marine Corps enlisted manpower levels. Models are developed for 44 representative population groups using Holt-Winters exponential smoothing, multiplicative decomposition, and Box-Jenkins autoregressive integrated moving average (ARIMA) forecasting methods. The forecasts are evaluated against actual, out-of-sample inventory levels using several goodness-of-fit indicators including Mean Absolute Error Rate (MAPE), Mean Absolute Error (MAE), and Sum of Squared Errors (SSE). Among the modeling techniques evaluated, the multiplicative decomposition performed the best overall and represents an improvement over the Marine Corps' current naïve forecasting method. This thesis recommends Marine Corps Systems Command, Total Force Information Technology Systems develop and introduce a multiplicative decomposition forecasting model into the Enlisted Staffing Goal Model. This forecasting technique should be implemented in phases, starting with the E-1 through E-4 population groups.

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EXECUTIVE SUMMARY

Accurately forecasting future manpower inventory levels in a fundamental prerequisite for the development of an effective and functional staffing plan. The Enlisted Staffing Goal Model (ESGM) generates a proposed enlisted-force staffing plan that mitigates the dilemma of how to optimally meet service-manning requirements within the constraints of the Marine Corps' limited inventory of assignable personnel. In order for the staffing plan to be effective, the model's inputs must be accurate. One of these inputs, the forecast inventory available for assignment, is the weakest component of the ESGM process. Developing and incorporating a more accurate manpower inventory forecasting model will result in a closer to optimal distribution of personnel to meet the Marine Corps staffing requirements and improve operational readiness.

A time series is a collection of observations in time. Although the values of individual observations cannot be predicted exactly, the distribution of stochastic time series observations commonly follows a discernable pattern. Statistical models can often describe these patterns. These models assume that the observations vary randomly about an underlying mean value that is a function of time. The time series may also be characterized by one or more behavioral components that may be isolated, modeled and incorporated into the forecast algorithm.

Univariate time series forecasting models forecasting models make predictions by extrapolating the past behavior of a single variable of interest. Forecasting is appropriate for stochastic time series data when the underlying causes of variation do not change significantly over time. The forecasting process consists of five steps. This heuristic method includes formulating the problem, obtaining data, selecting and applying forecasting methods, evaluating models, and using forecasts. Guided by this development framework, analysts can produce more accurate and efficient forecasts.

This thesis develops and evaluates manpower inventory forecasting models for 44 representative Marine Corps enlisted population groups using three univariate time series forecasting techniques. These methods are the Holt-Winters exponential smoothing,

multiplicative decomposition, and Box-Jenkins autoregressive integrated moving average forecasting models. Historical personnel strength levels obtained from the Marine Corps Total Force Data Warehouse comprise the observation sets employed in this thesis.

The Holt-Winters exponential smoothing method is a common univariate time series forecasting technique. This model smoothes irregular fluctuations by using weighted averages in an exponentially decreasing manner. The technique creates forecasts by estimating and extrapolating a linear trend while adjusting the data in each period by estimated seasonal indices.

Multiplicative decomposition is another effective method of forecasting univariate time series data. This technique is based on the concept that the underlying factors can be identified and isolated. These factors are trend, cycles, seasonality, and random variation. The analyst develops a model by first identifying and removing the component effects from the data. After these effects are isolated, the analyst creates a forecast by reassembling the components.

The Box-Jenkins technique is a sophisticated approach by which to analyze time series data and extrapolate a forecast. This methodology provides a framework for preparing data, selecting a model, and creating forecasts commonly referred to as autoregressive integrated moving average (ARIMA) forecasting. This technique assumes any given time series observation can be modeled as a function of its past values and current and past values of random errors.

Models developed in this study using the Holt-Winters exponential smoothing, the multiplicative decomposition, and the Box-Jenkins forecasting techniques are compared to the current forecasting method and each other applying a variety of statistical goodness-of-fit measurements. The evaluation techniques used are the Mean Absolute Percentage Error, the Sum of Squared Errors, and the Mean Absolute Error. Among the forecasting techniques evaluated, the multiplicative decomposition method performed best overall, and represents an improvement over the Marine Corp’s current forecasting method.

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I. INTRODUCTION

A. BACKGROUND

The purpose of the Marine Corps assignment process is to satisfy manpower readiness requirements through the best distribution of available personnel. Accurately forecasting future personnel inventory levels by rank and occupational specialty is a fundamental prerequisite for development of an effective and functional staffing plan.

> Manpower forecasting seeks to increase an organization’s human resource options and reduce the penalty costs of lost opportunity and lowered performance in managing those resources (Burack & Mathys, 1980).

The Enlisted Staffing Goal Model (ESGM) generates a proposed enlisted-force staffing plan that mitigates the dilemma of how to optimally meet service-manning requirements within the constraints of the Marine Corps’ limited assignable personnel inventory. In order for the staffing plan to be effective, the model’s inputs must be accurate. These inputs include manpower requirements, staffing policies, and projected available personnel inventory.

The ESGM’s forecasted manpower inventory is the model’s leading weakness. Currently this supply is represented by a snapshot of the existing personnel inventory captured six-months prior to the execution phase of the assignment process. This naïve forecasting method does not consider the myriad of factors that affect personnel inventory levels over time including promotions, reductions, reenlistments, extensions, deaths, MOS changes, and administrative discharges. This negatively impacts the utility of the staffing plan.

Without an accurate inventory-forecasting model, the ESGM currently produces unrealistic staffing goals that are frequently unachievable at the time of execution. This shortcoming significantly impairs the realization of the model’s primary objective – ameliorating the manpower system’s mismatch between requirements and the assignable population. Developing and incorporating a more accurate inventory forecasting method will result in a closer to optimal distribution of personnel to meet the Marine Corps’ manpower requirements and improve operational readiness.

B. PURPOSE

The purpose of this thesis is to improve the output of the Marine Corps Enlisted Staffing Goal Model by more accurately predicting the personnel resources available for assignment to future manpower requirements. The primary research question is the following: What univariate time series modeling technique is the most effective method to forecast Marine Corps enlisted personnel inventory levels by rank and occupational specialty, six and twelve months in the future?

C. SCOPE AND METHODOLGY

This thesis develops and evaluates manpower inventory forecasting models for 44 representative Marine Corps enlisted population groups using three univariate time series forecasting models. These techniques are the Holt-Winters exponential smoothing, multiplicative decomposition, and Box-Jenkins autoregressive integrated moving average forecasting methods. Historical personnel strength levels obtained from the Marine Corps Total Force Data Warehouse comprise the observation sets employed in this study. These time series contain 48 monthly observations starting in October 2001 and concluding in September 2005. The forecasts are evaluated against actual, out-of-sample inventory levels using several goodness-of-fit measures including Mean Absolute Error Rate, Sum of Squared Errors, and Mean Absolute Error.

D. BENEFITS OF THE STUDY

By developing and evaluating various manpower forecasting models, this study will provide the necessary information required to integrate a more effective enlisted personnel strength prediction into the Enlisted Staffing Goal Model. Incorporating this forecasting model will result in a closer to optimal distribution of the assignable inventory to meet the Marine Corps' manpower requirements and improve readiness.

E. ORGANIZATION

This introductory chapter provides the reader with the motivation, objective, and organization of this thesis. Chapter II offers a synopsis of time series analysis and the forecast model development process. Chapter III presents an overview of the Marine Corps Enlisted Staffing Goal Model. Chapter IV introduces the data used to develop the study's forecasting models. Chapters V through VII discuss the Holt-Winters exponential smoothing, multiplicative decomposition, and Box-Jenkins modeling

techniques, respectively, and demonstrate forecast development. Chapter VIII reviews model evaluation techniques and presents a comparison and analysis of the forecast results. The thesis concludes with recommendations and potential areas for further research.

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II. TIME-SERIES ANALYSIS AND FORECASTING

A. OVERVIEW

A time series is a sequence of observations obtained through measurements often recorded at equally spaced intervals. Often, time series data have characteristics that facilitate forecasting. These include seasonality, underlying trends, and relationships with past observations or other causal variables. Analysts can improve time series forecasts if they understand the nature of these components and identify the model that will best exploit the data's characteristics.

The purpose of this chapter is to provide a synopsis of time series analysis and forecasting. The first section discusses the characteristics of time series data. It reviews the common components useful in creating effective forecasts such as trend, seasonality, cyclical behavior, and irregular fluctuations. The chapter concludes with an introduction to time series forecasting and an overview of the forecasting model development process.

B. CHARACTERISTICS OF TIME-SERIES DATA

A time series is a "collection of observations made sequentially in time" (Chatfield, 1996). Examples are records of local daily rainfall levels, the quarterly U.S. Gross Domestic Product, and the monthly Marine Corps personnel strength for a particular rank and MOS. Time series analysis provides tools for choosing a representative model and producing forecasts.

There are two kinds of time series data:

- Continuous, where the data contain an observation at every instant of time, e.g., seismic activity recorded on a seismogram.
- Discrete, where the data contain observations taken at intervals, e.g., monthly crime figures.

Unless the data are purely random, observations in a time series are normally correlated and successive observations may be partly determined by past values (Chatfield, 1996). For example, the meteorological factors that affect the temperature on any given day are likely to exert some influence on the following day's weather. Thus, historical temperature observations are beneficial in forecasting future temperatures.

A time series is deterministic if it contains no random or probabilistic characteristics but proceeds in a fixed, predictable fashion (Chatfield, 1996). An example of a deterministic time series would be the data collected while conducting a classical physics experiment such as one demonstrating Newton's law of motion (Gujarati, 2003). More applicable to econometric applications are stochastic time series. Stochastic variables have indeterminate or random aspects. Although the values of individual observations cannot be predicted exactly, measuring the distribution of the observations may follow a predictable pattern. Statistical models can describe these patterns. These models assume that observations vary randomly about an underlying mean value that is a function of time. Time series data can also be characterized by one or more behavioral components: trend, seasonality, cyclical behavior, and random noise.

1. Trend Component

Trend is the general drift or tendency observed in a set of data over time. It is the underlying direction (an upward or downward tendency) and the degree of change in an observation set when consideration has been made for other components. Graphing a time series can be a useful and simple method of identifying the trend of a particular data set. Figure 1 indicates an upward trend of the U.S. Gross Domestic Product over a ten-year span. Analysts can also discern trends by dividing the data set into a number of ranges, and calculating the mean for each span. A consistent increase or decrease in the mean for the successive ranges indicates trend.

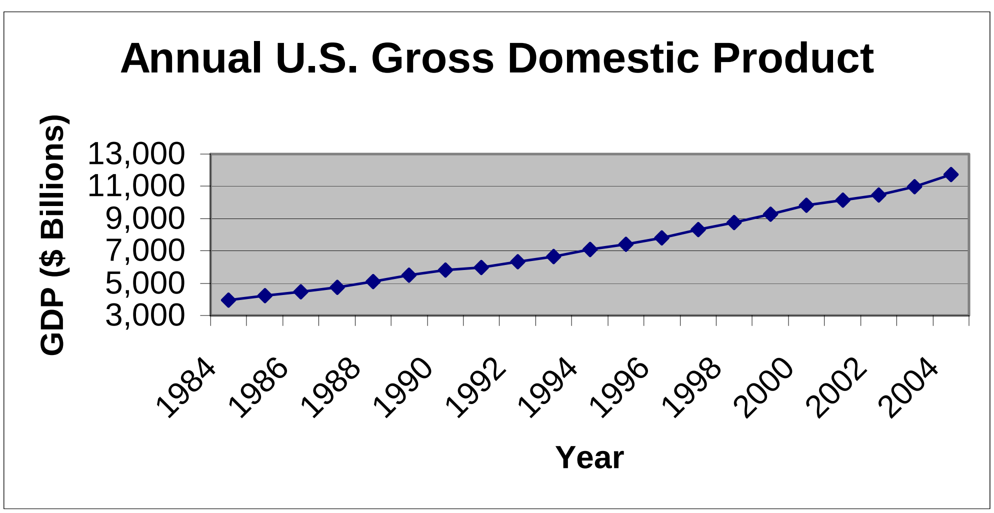

Figure 1. Annual U.S. Gross Domestic Product ($ billions) 1984-2004

Trends in business or economic series may be due to a growth or contraction process. Trends in service manpower levels may be attributed to external economic factors or shifts in policy due to technical innovation, downsizing, or an increased or decreased operational requirement for certain occupational specialties.

2. Seasonal Component

In time series data, the seasonal component is the element of variation in a data set that is dependent on the time of year. Seasonality is quite common in econometric time series. It is less common in engineering and scientific data. This component recurs annually, with possible variations in amplitude. Seasonality is attributable to the change of seasons and/or the timing of such events as holidays or the start or completion of the school term. For example, the cost of fresh produce, retail sales levels, average daily rainfall amounts, and unemployment figures all demonstrate seasonal variation. Figure 2 effectively illustrates the effect of seasonality on the California unemployment rate over a twenty-year period.

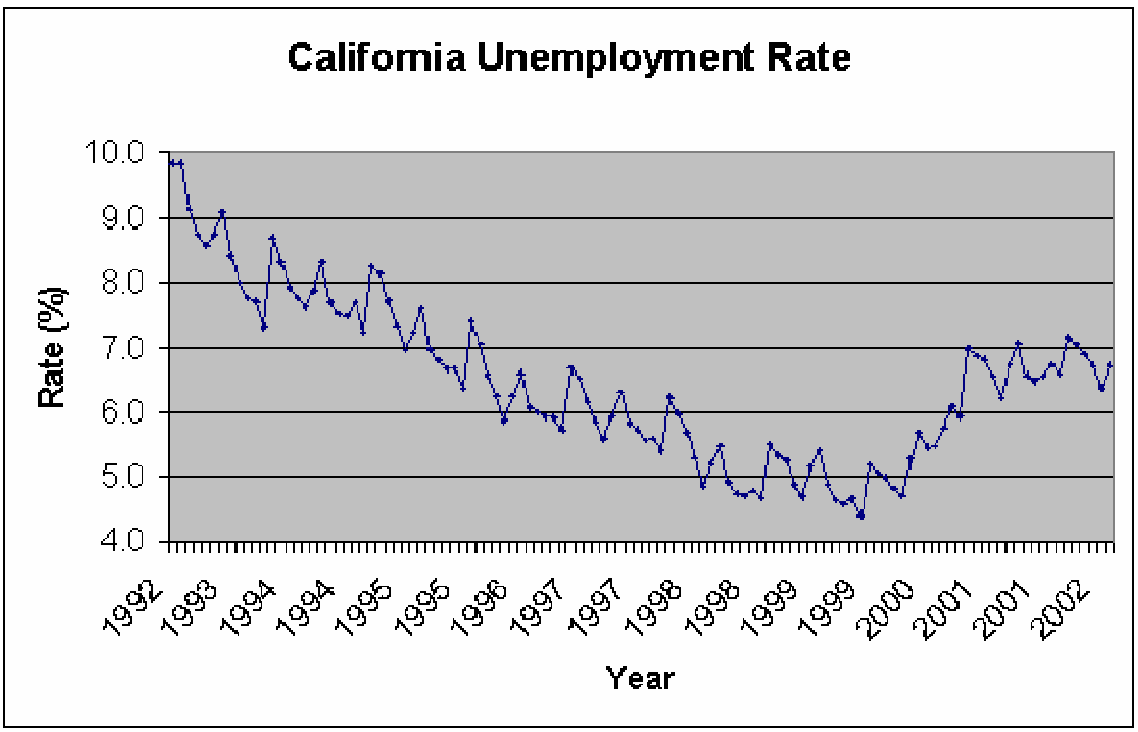

Figure 2. Monthly California Unemployment Rate

Incorporating seasonality in a forecast is useful when the time series has a discernible seasonal component. When the data contain a seasonal effect, it is useful to separate the seasonality from the other components in the time series. This enables the analyst to estimate and account for seasonal patterns.

3. Cyclical Component

Cyclical behavior describes any non-seasonal component that oscillates in a recognizable pattern. The 11-year sunspot cycle has been long recognized as naturally occurring cyclical activity. More ambiguous is the 5 to 7 year business cycle that a number of economists hypothesize influence global economic activity. If the data include a discernible cyclical component, the time series should span enough cycles to accurately model and forecast its effects (Yaffee, 2000). Cyclical behavior in which the oscillations extend over a very long period (such as 20 years) can often be accurately modeled as a trend for short-term forecasts (Chatfield, 1996).

4. Irregular Component

A significant component of stochastic time series data is irregular fluctuation. This random noise is what remains after the other components of the series (trend, seasonality, and cyclical behavior) are estimated and eliminated. It results from fluctuations in the series that are neither systematic nor predictable. While the irregular component typically has a modest impact on attempts to analyze, model and forecast time series data, it can sometimes have a significant effect. The effect of the 1973 OPEC (Organization of Petroleum Exporting Countries) oil embargo on the U.S. economy is an example of the substantial consequences irregular fluctuations can occasionally inflict on time series data.

C. TIME-SERIES FORECASTING

Univariate time series forecasting models make predictions by extrapolating the past behavior of the values of a particular single variable of interest (Moore & Weatherford, 2001). Successive observations in econometric time series are normally not independent and predictions may be made from previous observations (Chatfield, 1996). While exact forecasts are possible with deterministic time series, forecasts of stochastic time series are limited to “conditional statements about the future based on specific assumptions” (Chatfield, 1996). According to Armstrong (2001), “the basic

assumption is that the variable will continue in the future as it has behaved in the past." Specifically, time series forecasts are appropriate for stochastic data where the underlying causes of variation – trend, cyclical behavior, seasonality, and irregular fluctuations – do not change significantly in time (Jenson, 2004). Hence, modeling is often more appropriate for short-term than for long-term forecasting.

1. The Forecasting Model Development Process

In his book *Forecasting Principles*, Armstrong (2001) proposes a heuristic forecasting process. Guided by the five general steps of Armstrong's development framework, analysts can produce more accurate and efficient forecasts. These steps are graphically depicted in Figure 3 and include formulating the problem, obtaining information, selecting and applying forecasting methods, evaluating models, and using the forecasts. The remainder of this chapter summarizes Armstrong's forecasting principles as they apply to this thesis.

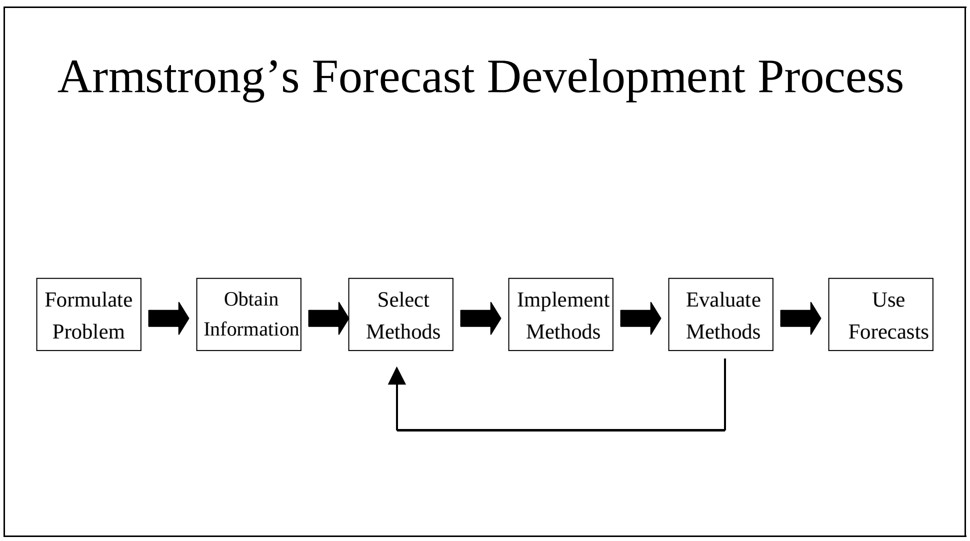

Figure 3. Forecast Development Process (From: Armstrong, 2001)

2. Formulate the Problem

The first step in Armstrong's (2001) forecast development process is to formulate the problem. The analyst must clarify the user's objectives for the forecast and the decisions associated with these particular goals. Of particular note is the initial assessment as to whether a practical forecast is possible for a particular problem. Forecasting should be avoided if previous studies indicate that the scenario is unlikely to yield a beneficial result. Additionally the analyst should structure the problem

appropriately in order to forecast effectively and produce results in a format constructive for the user. This might include decomposing the underlying factors affecting the situation, identification of significant relationships between causal variables, and recognition of trends or other components of time series data.

3. Obtaining Information

After appropriately formulating the problem, the next step in Armstrong's (2001) forecast development process is the identification, collection, and preparation of data. The analyst can employ theory and previous research to identify potential explanatory variables. These data should correspond to the forecasting problem and be objective in nature. Often the best predictor of future behavior, given a particular scenario, is past behavior. Once a source is identified, the forecaster must collect all pertinent, suitable and reliable data and prepare it for the forecasting process. Preparing the data for modeling includes scrubbing the series, adjusting for errors and missing observations where required and employing statistical techniques to adjust for unsystematic past events. A hurricane would be an example of an unsystematic past event when analyzing regional retail sales time series data. Some models may also require data transformation in order to render the series stationary in mean, variance, and autocovariance (Yaffee, 2000).

4. Select and Implement Forecasting Methods

Often many methods are applicable to a particular forecasting situation. The forecaster should list and consider all of the selection criteria before selecting a forecasting method. Although accuracy is an essential measure of effectiveness, other criteria such as cost, development time, expertise available, ease of use, and ease of implementation are important considerations as well. Quantitative forecasting methods tend to be less subjective than qualitative techniques and are suitable for occasions when germane data are readily available. If feasible, the analyst should evaluate the forecasts of several methods under comparable conditions noting the accuracy, cost, and usability of each technique.

Occam's Razor states that given two equally predictive theories, an individual should choose the simplest. Thus, to improve accuracy, assist user comprehension,

decrease errors, and reduce expenses, the forecasting model should be as uncomplicated as practical – using few variables and simple relationships between among them.

Additionally, the model should be tailored to the requisite planning horizon. Short-term forecasts should place added weight on the most recent observations. Distant forecast lengths should emphasize applicable long-term trends and cycles. Although there are no established guidelines regarding minimum sample size, the series should contain sufficient observations for accurate parameter estimation. If the series contains seasonality or other cycles, the data should span enough oscillations to effectively model the cyclical behavior (Yaffee, 2000). Thus seasonal or cyclical processes "require more observations than non-seasonal ones" (Yaffee, 2000). In addition to Armstrong's (2001) criteria for model selection and implementation above, Chatfield (1996) recommends choosing methods that account for the number of series to be forecast and the resources required per forecast, suggesting that "univariate forecasts are particularly suitable when there are large numbers of series to be forecast."

5. Evaluate Methods

Methods for evaluating forecast models are based on conventional analytical procedures. Armstrong (2001) suggests evaluating at least two "reasonable" forecasting techniques including the current method if applicable. The analyst should evaluate the models under situations that match the forecasting problem. One effective method is to back-forecast employing older observations in the time series comparing the forecast fit against the actual values of recent, out-of-sample observations (Wooldridge, 2003). When evaluating the effectiveness of forecast methods across several series, the analyst should utilize error measurements that consider the scale of the observations (Armstrong, 2001). This measurement must not be subject to distortion due to a disparity in series observation value magnitudes (thus weighting some samples more heavily). A measurement that incorporates percentage error is appropriate for these circumstances.

If the forecasting methods evaluated are not adequate for the problem, the analyst should consider another technique. Finally, the analyst and the end user should consider the costs and benefits of each model evaluated in order to select the most appropriate method.

6. Use Forecasts

After selecting a model, the analyst should present the forecast and the supporting data in a straightforward and comprehensible format, such as a graph. The documentation should provide a complete and clear explanation of the data, methods, and assumptions. The forecaster should review the forecast accuracy periodically, updating the parameters of the model as the data or situation changes and actively seek user feedback.

III. THE ENLISTED STAFFING GOAL MODEL

A. OVERVIEW

The Enlisted Staffing Goal Model (ESGM) generates a proposed enlisted force-staffing plan that resolves the dilemma of how to optimally meet service-manning requirements within the constraints of the Marine Corps' limited assignable personnel inventory. The primary focus of this chapter is to provide an overview of the ESGM. First, the model's purpose and utility are examined. Next, the ESGM is examined from a system perspective, including input, processes and output. Finally, the chapter concludes with an examination of effectiveness of the model and identification of a significant limitation that impedes the ESGM's value.

B. FUNCTIONALITY AND USE

The purpose of the Marine Corps' inventory assignment process is to make the best distribution of current assignable inventory to meet the Marine Corps authorized strength requirement according to current staffing precedence policies. The foundations of personnel requirements are established in the individual unit Tables of Organization (T/O's). These documents are based upon each unit's defined mission. The sum of the service's T/O's typically exceed the number of personnel authorized by Congress and available for assignment. Currently, the Marine Corps has over 177,000 active duty service members. Of these, approximately 144,000 Marines are available to staff 154,000 structured billet requirements. The manning plan developed annually by Headquarters Marine Corps is called the Authorized Strength Report (ASR) and is developed based on T/O's, Congressional authorization, operational requirements and service policies. The quantities and skills of the existing inventory of Marines are not considered when producing the ASR. Thus there is routinely a mismatch between the actual number of billets authorized and the assignable population. The Manpower and Systems Support Section of the Headquarters Marine Corps' Enlisted Assignments Branch employs the ESGM to manage the predicament of how to allocate the assignable personnel inventory to the billet requirements with the best skill fit (as defined by rank and military occupational specialty (MOS)).

The purpose of the ESGM is to produce a set of targets called staffing goals, which represent a billet requirement that can be filled using the available inventory. This staffing plan is provided to subordinate organizations as guidance from Headquarters Marine Corps indicating which billets the Enlisted Assignments Branch intends to fill at each unit annually. Staffing goals are perceived as a commitment and frequently described metaphorically as a “debt” which the Enlisted Assignment branch “repays” to the operating force and supporting commands each staffing period using the assignable inventory as “currency” (HQMC, MMEA-5, 2005).

C. ESGM STRUCTURE AND PROCESSING OVERVIEW

The staffing goals also guide the individual detailers in the execution of the assignment. However, in order for the staffing goals to be practical and functional, the inputs must be effective and accurate. The ESGM’s inputs include requirements, assignment and staffing policies, and projected available personnel inventory.

Manpower requirements input is provided in the form of the ASR which constrains the combat-based, Marine Corps T/O personnel requirements by (a) the manpower levels authorized by Congress and (b) the “Transients, Trainees, Prisoners and Patients” (T2P2) account. T2P2 estimates the number of Marines unavailable for assignment annually for time spent executing orders, training in excess of 20 weeks, hospitalization for greater than 30 days, or incarceration for between 30 and 180 days. The ASR identifies billet requirements by unit, pay grade, MOS, and number authorized.

The model also integrates assignment and staffing policies to balance the insufficient supply with the demand. These are a set of business rules and constraints that optimize the “fill” and “fit” in accordance with the Commandant’s priorities and billet requirements. The “fill” is determined by the staffing precedence order (Marine Corps Order 5320.12E, Precedence Levels for Manning and Staffing) that specifies which particular units will be staffed at 100% of T/O level (Excepted Commands), which units will be staffed at 95% of T/O level (Priority Commands), and which units will be staffed with the remaining inventory on a proportionate share basis (Pro-Share Commands). The “fit” is determined by a set of restrictions and substitution rules, which may permit grade and MOS substitutions or establish gender restrictions for particular billets.

The third input, the forecasted assignable manpower inventory, is provided in the form of an extract of personnel information from the Marine Corps Total Force System Operational Data Store Enterprise (MCTFS/ODSE). The ESGM processes the data and creates a forecasted inventory of assignable personnel based on the specific Marines' remaining service obligations. Specifically, the model considers an individual eligible to fill a staffing goal for his or her current rank and MOS if his or her end of active service (EAS) date exceeds the current date by six months or more.

The optimization process has two objectives: maximize the fill of billets and maximize the fit of individuals to the billets. The ESGM maximizes the fill while honoring the policy-defined priority. Billets with higher priority are filled before lower priority billets. Within any priority level, shortages are shared in proportion to the target if allowed by "fit" rules. The result of this process is a set of feasible quotas, which the ESGM then uses to maximize the "fit" of individuals. The ESGM endeavors to allocate individuals within the fill constraints to the highest level of desirability as measured by occupational specialty. Finally, the model attempts to allocate individuals with a perfect grade match proportionately to the targets.

Included among the outputs generated by the ESGM are a database containing all information required to run the Enlisted Assignment Model (EAM), data posted to the Monitor Assignment Support System (MASS), and the Command Staffing Report (CSR). The CSR is a critical document used by both the assignment monitors and individual commands. It specifies which T/O billets are authorized and which billets received a staffing goal for a particular staffing period.

D. ACCURACY

The ESGM's forecasted manpower inventory is a major weakness of the model. Currently this supply is represented by a snapshot of the existing personnel inventory six months prior to the execution phase of the assignment process. Other than removing those Marines with an approved separation date, the model does not forecast the projected inventory at the time of execution. This naïve forecasting method does not consider the myriad of factors that affect personnel inventory levels over time including promotions, reductions, reenlistments, extensions, deaths, MOS changes, and administrative discharges. This affects the utility of the staffing plan.

Without an effectual inventory-forecasting model, the ESGM currently produces unrealistic staffing goals that are unachievable at the time of execution. This shortcoming significantly impairs the realization of the model's primary objective – ameliorating the manpower system's mismatch between requirements and the assignable population. Developing and incorporating a more accurate inventory forecasting method will result in a closer to optimal distribution of personnel to meet the Marine Corps' manpower requirements.

IV. DESCRIPTION OF DATA

A. MONTHLY MANPOWER INVENTORY LEVELS

This thesis develops and evaluates models for the 44 MOS/paygrade combinations detailed in Table 1. The Marine Corps Enlisted Assignments Branch considers these groups a representative sample of the overall enlisted Marine Corps population. The Appendix summarizes the time series forecasts constructed for each population. Monthly historical personnel inventory level time series were obtained from the Marine Corps Total Force Data Warehouse (TFDW).

Marine Corps Manpower
Inventory Time Series Analyzed

• **E5 0211**	• **E7 0369**	• **E8 6019**
• **E6 0211**	• **E8 0369**	• **E9 6019**
• **E7 0211**	• **E9 0369**	• **E1-E3 6092**
• **E8 0211**	• **E5 2336**	• **E4 6092**
• **E1-E3 0311**	• **E6 2336**	• **E5 6092**
• **E4 0311**	• **E7 2336**	• **E6 6092**
• **E5 0311**	• **E8 2336**	• **E7 6092**
• **E1-E3 0321**	• **E9 2336**	• **E1-E3 7257**
• **E4 0321**	• **E1-E3 5811**	• **E4 7257**
• **E5 0321**	• **E4 5811**	• **E5 7257**
• **E6 0321**	• **E5 5811**	• **E6 7257**
• **E7 0321**	• **E6 5811**	• **E7 7257**
• **E8 0321**	• **E7 5811**	• **E8 7291**
• **E9 0321**	• **E8 5811**	• **E9 7291**
• **E6 0369**	• **E9 5811**	

Table 1. MOS/Paygrade Population Inventory Time Series Analyzed

Each time series contains monthly manpower inventory levels with 48 observations starting in October 2001 and concluding in September 2005. The modeling techniques evaluated in this thesis employ the first 36 observations to develop forecasts. The final 12 observations in each series are reserved for evaluating each model's forecast accuracy with respect to actual inventory levels. The period includes a 6-month interval in which the Marine Corps enacted a stop-loss policy in response to combat operations in

Iraq. This policy involuntarily extended Marines on active duty, substantially increasing the inventory levels for several population groups from February 2003 until July 2003.

B. E-5 0311 TIME SERIES

The following chapters refer to the 0311 E-5 (Rifleman sergeant) time series in order to demonstrate the development of the exponential smoothing, multiplicative decomposition, and Box-Jenkins forecasting models. Table 2 provides the monthly inventory levels for the 0311 E-5 population group retrieved from the TFDW. The graphical representation of the data provided in Figure 5 demonstrates the volatility of the inventory levels and highlights the time series' significant irregular component resulting from stop-loss.

USMC 0311 E-5 Monthly End Strength, Oct 01 - Sept 05					
Period	Month	Inventory	Period	Month	Inventory
1	10/1/01	1835	25	10/1/03	1895
2	11/1/01	1840	26	11/1/03	1859
3	12/1/01	1833	27	12/1/03	1807
4	1/1/02	1823	28	1/1/04	1790
5	2/1/02	1833	29	2/1/04	1868
6	3/1/02	1857	30	3/1/04	1889
7	4/1/02	1902	31	4/1/04	1873
8	5/1/02	1877	32	5/1/04	1898
9	6/1/02	1866	33	6/1/04	1856
10	7/1/02	1809	34	7/1/04	1842
11	8/1/02	1849	35	8/1/04	1869
12	9/1/02	1850	36	9/1/04	1837
13	10/1/02	1765	37	10/1/04	1822
14	11/1/02	1908	38	11/1/04	1825
15	12/1/02	1964	39	12/1/04	1838
16	1/1/03	1943	40	1/1/05	1816
17	2/1/03	2074	41	2/1/05	1877
18	3/1/03	2126	42	3/1/05	1869
19	4/1/03	2156	43	4/1/05	1853
20	5/1/03	2191	44	5/1/05	1871
21	6/1/03	2214	45	6/1/05	1858
22	7/1/03	2076	46	7/1/05	1857
23	8/1/03	1947	47	8/1/05	1876
24	9/1/03	1934	48	9/1/05	1880

Table 2. USMC 0311 E-5 Monthly End Strength, Oct 01 – Sept 05

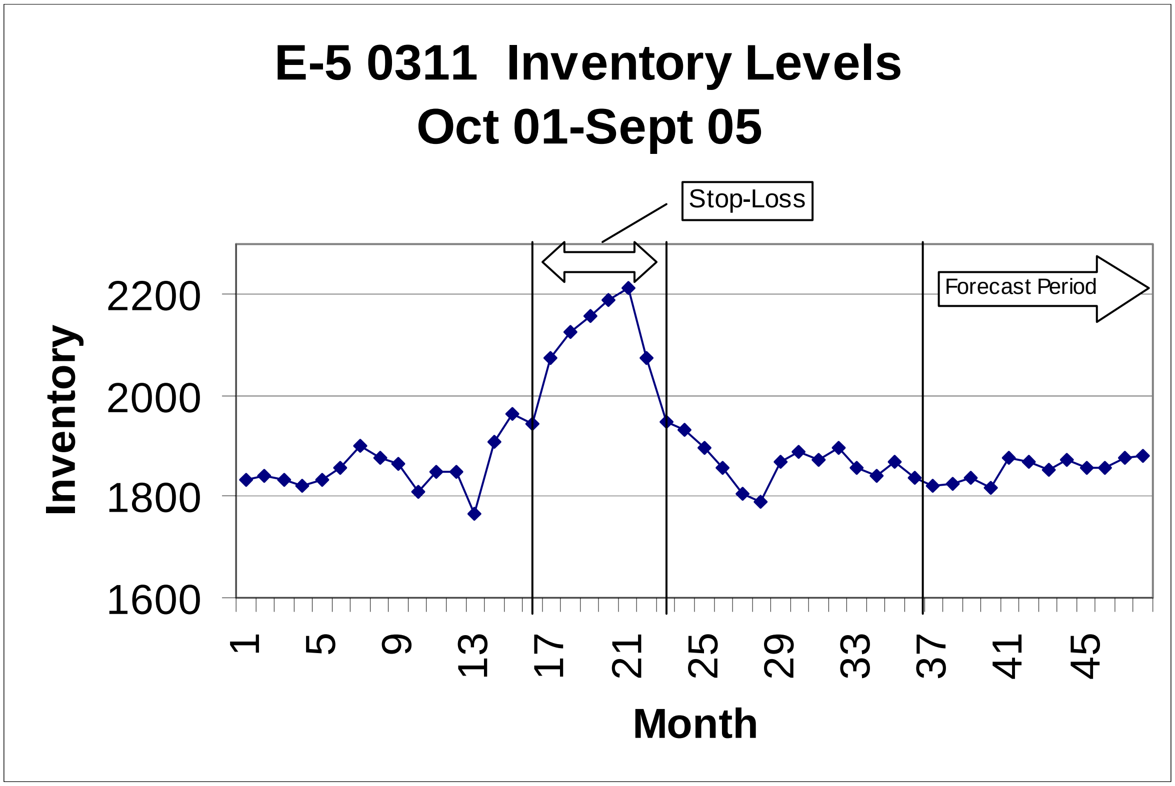

Figure 4. 0311 E-5 Inventory Levels Oct 01 – Sept 05

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V. EXPONENTIAL SMOOTHING FORECASTING

A. OVERVIEW

Exponential smoothing is a common univariate time series forecasting technique. This method smoothes irregular fluctuations by using weighted averages of time series observations in an exponentially decreasing manner. The older an observation is, the less influence it has on the forecast. More advanced exponential smoothing models endeavor to identify and model time series components and incorporate them into the forecast (Yaffee, 2001). Exponential smoothing is appropriate for automatic, short-term forecasting of frequently used data where the underlying causes of variation do not change markedly over time.

This chapter reviews exponential smoothing forecasting techniques. An examination of the simple exponential smoothing model provides the technical foundation for the remainder of the chapter. The following sections discuss linear and seasonal exponential smoothing methods. The chapter concludes by demonstrating the application of the exponential smoothing forecasting process. It employs the Holt-Winters technique to forecast manpower inventory levels for the E-5, 0311 Marine Corps enlisted population group introduced in Chapter IV.

B. SIMPLE EXPONENTIAL SMOOTHING

Exponential smoothing in its most basic form is appropriate for forecasting time series with no seasonality or trend components (Chatfield, 1996). It provides a starting point for the more advanced smoothing techniques evaluated in this thesis. Exponential smoothing places more weight on the most recent observations and exponentially less weight on older values. To examine this method in detail, consider a time series $\{x_1, x_2, \ldots, x_t\}$ where x_t is the current observation. The analyst can extrapolate the value at one time period in the future, x_{t+1}, as a weighted sum of the previous observations.

Consider the formula:

$$\hat{x}_{t+1} = c_0 x_t + c_1 x_{t-1} + c_2 x_{t-2} + \ldots \tag{5.1}$$

where c_i are weights. In Equation (5.1), the weights decay exponentially from the most recent to the most distant observation. These weights sum to one, and the proportion of the latest observation taken is the constant α, commonly referred to as the damping factor or the smoothing constant:

$$\begin{aligned} & c_i = \alpha(1-\alpha) \\ where: \ & i = 1, 2, \ldots, \\ and: \ & 0 \le \alpha \le 1. \end{aligned} \tag{5.2}$$

Thus, Equation (5.1) becomes:

$$\hat{x}_{t+1} = \alpha x_t + \alpha (1-\alpha) x_{t-1} + \alpha (1-\alpha)^2 x_{t-2} + \ldots \tag{5.3}$$

This formula presumes an unlimited number of observations when in actuality the forecast requires only a finite number (Chatfield, 1996). Therefore, Equation (5.3) is restated in the recurrence form as:

$$\begin{aligned} \hat{x}_{t+1} &= \alpha x_t + (1-\alpha)[\alpha x_{t-1} + \alpha(1-\alpha) x_{t-2} + \ldots] \\ &= \alpha x_t + (1-\alpha)\hat{x}_{t-1}. \end{aligned} \tag{5.4}$$

This equation requires an estimate of $\hat{x}_1$. A straightforward and effective option is $\hat{x}_1 = x_1$. Equation (5.4) can be rewritten to incorporate error correction:

$$\hat{x}_{t+1} = \hat{x}_t + \alpha(x_t - \hat{x}_t) \tag{5.5}$$

where, beginning with $t = 1$, the forecast for the next period, $\hat{x}_{t+1}$, equals the sum of the forecast for the current time period, $\hat{x}_t$, and the weighted error of the current period's forecast, $\alpha(x_t - \hat{x}_t)$. Forecasts are computed in this manner for all observations in the time series.

The choice of the smoothing constant, α, impacts the accuracy of the forecast significantly. Alpha is calculated to minimize the sum of discrepancies between the time series's actual observations and the forecast values. This computation takes the form of an optimization program:

$$\begin{aligned} &Minimize: \sum_{i=1}^{t}(x_i - \hat{x}_i)^2 \\ &Subject\ to: \hat{x}_{i+1} = \hat{x}_i + \alpha(x_i - \hat{x}_i), \\ &\qquad\qquad i = 1, 2, \ldots, t-1, \\ &and \qquad\quad 0 \le \alpha \le 1. \end{aligned} \tag{5.6}$$

C. HOLT'S LINEAR EXPONENTIAL SMOOTHING

Simple exponential smoothing does not account for the trend component common in many time series. By estimating and extrapolating the series' linear trend, the Holt linear exponential smoothing method often produces a forecast more accurate than simple exponential smoothing. The model's intercept and slope are calculated from weighted averages of previous observations, using separate smoothing constants for each. These constants are denoted by α and β respectively. The equations required to calculate the level, L_i, and linear trend, T_i, are:

$$\begin{aligned} &L_{t+1} = \alpha x_t + (1-\alpha)(L_t + T_t) \\ &where: \ 0 \le \alpha \le 1 \end{aligned} \tag{5.7}$$

$$\begin{aligned} &T_{t+1} = \beta(L_{t+1} - L_t) + (1-\beta)T_t \\ &where: \ 0 \le \beta \le 1. \end{aligned} \tag{5.8}$$

Initial estimates are required for L_1 and T_1. Simple options are:

$$L_1 = x_1 \tag{5.9}$$

$$T_1 = x_2 - x_1. \tag{5.10}$$

Thus, the forecast for the next time period is calculated using Equation (5.11), below. Predictions for i time periods in the future are forecast with Equation (5.12).

$$\hat{x}_{t+1} = L_{t+1} + T_{t+1} \tag{5.11}$$

$$\hat{x}_{t+i} = L_{t+1} + (i)T_{t+1} \tag{5.12}$$

As with the simple exponential smoothing model, the values of the damping factors, α and β, are important determinates of the model's accuracy. Minimizing the disparities between the model's actual and predicted values will optimize these constants. Equation (5.13) summarizes this optimization program.

$$
\begin{aligned}
&Minimize: \sum_{i=1}^{t} (x_i - \hat{x}_i)^2 \\
&Subject\ to: L_{i+1} = \alpha x_i + (1-\alpha)(L_i + T_i), \\
&\qquad T_{i+1} = \beta(L_{i+1} - L_i) + (1-\beta)T_i, \\
&\qquad \hat{x}_{i+1} = L_{i+1} + T_{i+1}, \\
&\qquad i = 1, 2, \ldots, t-1, \\
&\qquad 0 \le \alpha \le 1, \\
&and \qquad 0 \le \beta \le 1
\end{aligned}
\tag{5.13}
$$

D. HOLT-WINTERS SEASONAL EXPONENTIAL SMOOTHING

The Holt-Winters forecasting technique is an extension of linear exponential smoothing which accommodates a seasonal component present in a time series. This model forecasts by estimating and extrapolating a linear trend while adjusting the data in each period by estimated seasonal indices. The intercept, slope and seasonality are computed using weighted averages of previous observation values, applying separate smoothing constants to each. These parameters are denoted α, β, and γ respectively.

Level, trend and seasonality are designated L_t, T_t, and S_t at time *t*, while the subscript *s* is the periodicity of the seasonality. Therefore, if the time series consists of monthly observations, *s* = 12. If the series is comprised of quarterly observations, *s* = 4. Equation (5.7) is modified to accommodate seasonality in the following manner[1]:

$$
\begin{aligned}
&L_{t+1} = \alpha(x_t / S_{t-s}) + (1-\alpha)(L_t + T_t) \\
&where: \ 0 \le \alpha \le 1.
\end{aligned}
\tag{5.14}
$$

This calculation requires the extrapolation of T_i, L_i and S_i for the first complete cycle of seasonality – the first twelve observations for monthly time series data. Equations (5.15) (5.16) and (5.17) provide straightforward methods of estimation.

$$T_i = x_{i+1} - x_i \tag{5.15}$$

$$L_{t-s} = \frac{x_1 + x_2 + \ldots + x_s}{s} \tag{5.16}$$

$$S_{t-s} = x_{t-s} - L_{t-s} \tag{5.17}$$

1 This thesis describes the multiplicative forecasting method. For a description of the additive method, see Yaffee (2001).

$$where: i = 1, 2, \ldots, s$$

Trend is computed in the same fashion as in linear exponential smoothing:

$$\begin{aligned} &T_{t+1} = \beta(L_{t+1} - L_t) + (1-\beta)T_t \\ &where:\ 0 \le \beta \le 1. \end{aligned} \tag{5.18}$$

After extrapolating the values, the analyst can compute the forecast for the next time period:

$$\hat{x}_{t+1} = (L_t + T_t)S_{t+1-s}. \tag{5.19}$$

Seasonality is updated after observing the actual value of x_{t+1}:

$$\begin{aligned} &S_{t+1} = \gamma(x_{t+1} / L_{t+1}) + (1-\gamma)S_{t+1-s} \\ &where: 0 \le \gamma \le 1. \end{aligned} \tag{5.20}$$

Forecasts for time periods beyond the next are calculated in the following manner:

$$\begin{aligned} \hat{x}_{t+2} &= (L_{t+1} + 2T_{t+1})S_{t+2-s} \\ \hat{x}_{t+3} &= (L_{t+1} + 3T_{t+1})S_{t+3-s} \\ &\vdots \\ \hat{x}_{t+s} &= (L_{t+1} + sT_{t+1})S_t \\ \hat{x}_{t+s+1} &= (L_{t+1} + (s+1)T_{t+1})S_{t+1} \\ \hat{x}_{t+s+2} &= (L_{t+1} + (s+2)T_{t+1})S_{t+2-s} \\ &\vdots \end{aligned} \tag{5.21}$$

The smoothing constants are calculated via an optimization function similar to the simple and linear exponential smoothing models:

$$\begin{aligned} &Minimize: \sum_{i=1}^{t} (x_i - \hat{x}_i)^2 \\ &Subject\ to: L_{i+1} = \alpha(x_i / S_{t-s}) + (1-\alpha)(L_i + T_i), \\ &\qquad T_{i+1} = \beta(L_{i+1} - L_i) + (1-\beta)T_i, \\ &\qquad S_{i+1} = \gamma(x_{i+1} / L_{i+1})S_{t+1-s}, \\ &\qquad \hat{x}_{i+1} = (L_{i+1} + T_{i+1})S_{t+1-s}, \\ &\qquad i = s+1, s+2, \ldots t-1, \\ &\qquad 0 \le \alpha \le 1, \\ &\qquad 0 \le \beta \le 1, \\ &and \qquad 0 \le \gamma \le 1. \end{aligned} \tag{5.22}$$

E. MANPOWER FORECASTS WITH HOLT-WINTERS

Holt-Winters exponential smoothing techniques are applicable to forecasting manpower inventory levels for many Marine Corps enlisted population groups. This section demonstrates the process to predict E-5 0311 inventory levels six and twelve months in the future.

This time series contains monthly observations; therefore the periodicity of seasonality is twelve, i.e. $s = 12$. Only the first 36 observations will be used to create the forecast. The final twelve observations will be reserved to evaluate the forecast accuracy and are considered "out of sample" for the model. Therefore, for the purposes of the forecast, the most recent observation is period 36, the inventory level on September 1, 2004. Consequently, t = 36 and the current observation is denoted x_{36}.

Level, trend, and seasonality for the first twelve observations, $\{x_1, x_2, \ldots, x_{12}\}$, are computed using Equations (5.9), (5.16), and (5.17). These figures are presented in Table 3.

Period	Inventory	Level	Trend	Seasonality
1	1835	1835	5	1.0000
2	1840	1840	-7	1.0000
3	1833	1833	-10	1.0000
4	1823	1823	10	1.0000
5	1833	1833	24	1.0000
6	1857	1857	45	1.0000
7	1902	1902	-25	1.0000
8	1877	1877	-11	1.0000
9	1866	1866	-57	1.0000
10	1809	1809	40	1.0000
11	1849	1849	1	1.0000
12	1850	1850	-85	1.0000

Table 3. Level, Trend & Seasonality Calculations; $\{x_1, x_2, \ldots, x_{12}\}$

L_{13} and T_{13} are computed after observing x_{12} using Equations (5.14) and (5.18). At this point, the analyst has sufficient information to estimate $\hat{x}_{13}$, which is the forecast inventory level for October 1, 2002. Using Equation (5.19), the estimate, $\hat{x}_{13}$, is given by $(L_{13}+T_{13})*S_1$. After observing the actual value of x_{13}, the forecaster must update the seasonal effect for October, S_{13}, with Equation (5.20). This process is repeated for each

remaining observation in the time series, $\{x_{13}, x_{14}, \ldots, x_{36}\}$. These calculations are presented in Table 4.

Period	Inventory	Level	Trend	Seasonality	Prediction
13	1765	1848	-45.9550	0.9961	1802.1291
14	1908	1766	-63.0104	1.0070	1702.8265
15	1964	1903	31.2366	1.0028	1934.6121
16	1943	1963	44.7360	0.9991	2008.0736
17	2074	1944	14.8443	1.0058	1959.311
18	2126	2071	67.5270	1.0023	2138.9419
19	2156	2126	61.5821	1.0012	2187.8738
20	2191	2157	46.9408	1.0014	2203.6592
21	2214	2191	41.1257	1.0009	2232.4111
22	2076	2214	32.6686	0.9945	2247.0835
23	1947	2080	-45.9191	0.9944	2033.937
24	1934	1949	-85.8539	0.9993	1863.1056
25	1895	1932	-53.2884	0.9947	1871.7358
26	1859	1902	-42.5598	1.0044	1872.4567
27	1807	1846	-48.6980	1.0007	1802.6242
28	1790	1802	-46.6935	0.9986	1753.6077
29	1868	1791	-29.9615	1.0091	1771.0826
30	1889	1855	14.3003	1.0037	1873.6255
31	1873	1884	21.3463	1.0006	1907.9886
32	1898	1872	5.2938	1.0025	1879.4042
33	1856	1895	13.8240	0.9990	1910.5023
34	1842	1856	-11.1891	0.9944	1834.2913
35	1869	1852	-7.6287	0.9957	1834.0183
36	1837	1879	8.5304	0.9975	1885.9574

Table 4. Level, Trend, Seasonality & Forecast Calculations; $\{x_{13}, x_{14}, \ldots, x_{36}\}$

The forecaster should now optimize the smoothing constants by minimizing the disparities between the model's predicted values and the time series' actual observations. The optimization function specified in Equation (5.22) is easily accomplished with the Microsoft Excel Solver plug-in. The forecasting problem's optimized smoothing constants are presented in Table 5. The small value for gamma is indicative of the time series's nominal seasonal component.

Alpha	0.9774
Beta	0.46994
Gamma	0.08733

Table 5. Optimized Smoothing Constants

Using Equation (5.21), the analyst can now produce forecasts. The formulas for six and twelve month forecasts are presented as Equations (5.23) and (5.24) respectively,

below. Table 6 presents the predicted inventory level for each forecast period, the observed (actual) inventory level, and the absolute percentage error. The absolute percentage error, described in Equation (5.25), is one of many possible measures of forecast effectiveness. Figure 6 provides a graphical presentation of the observed and forecast inventory levels for each period.

$$\hat{x}_{42} = (L_{37} + 6T_{37})S_{30} \tag{5.23}$$

$$\hat{x}_{48} = (L_{37} + 12T_{37})S_{36} \tag{5.24}$$

$$APE = \frac{\left|(x_i - \hat{x}_i)\right|}{x_i} * 100 \tag{5.25}$$

Holt Winters Exponential Smoothing Forecast Results			
Forecast Period	Actual Inventory	Predicted Inventory	Absolute Percentage Error
6-Month	1869	1762	5.72%
12-Month	1880	1667	11.33%

Table 6. E-5 0311 Holt Winter's Exponential Smoothing Forecast Results

F. CHAPTER SUMMARY

Exponential smoothing is a common technique used to extrapolate a forecast from a historical time series. This method considers the entire past in the model, but weighs recent observations more heavily than older observations. Simple exponential smoothing models are appropriate for time series with nominal trend and seasonality components. Holt's linear smoothing technique isolates and models the series's trend, often producing a more accurate forecast than simple exponential smoothing. The Holt-Winters method is an extension of linear exponential smoothing which accommodates a seasonal component present in a time series. These models are appropriate for automatic, short-term forecasting of frequently used data where the underlying causes of variation are not changing markedly in time.

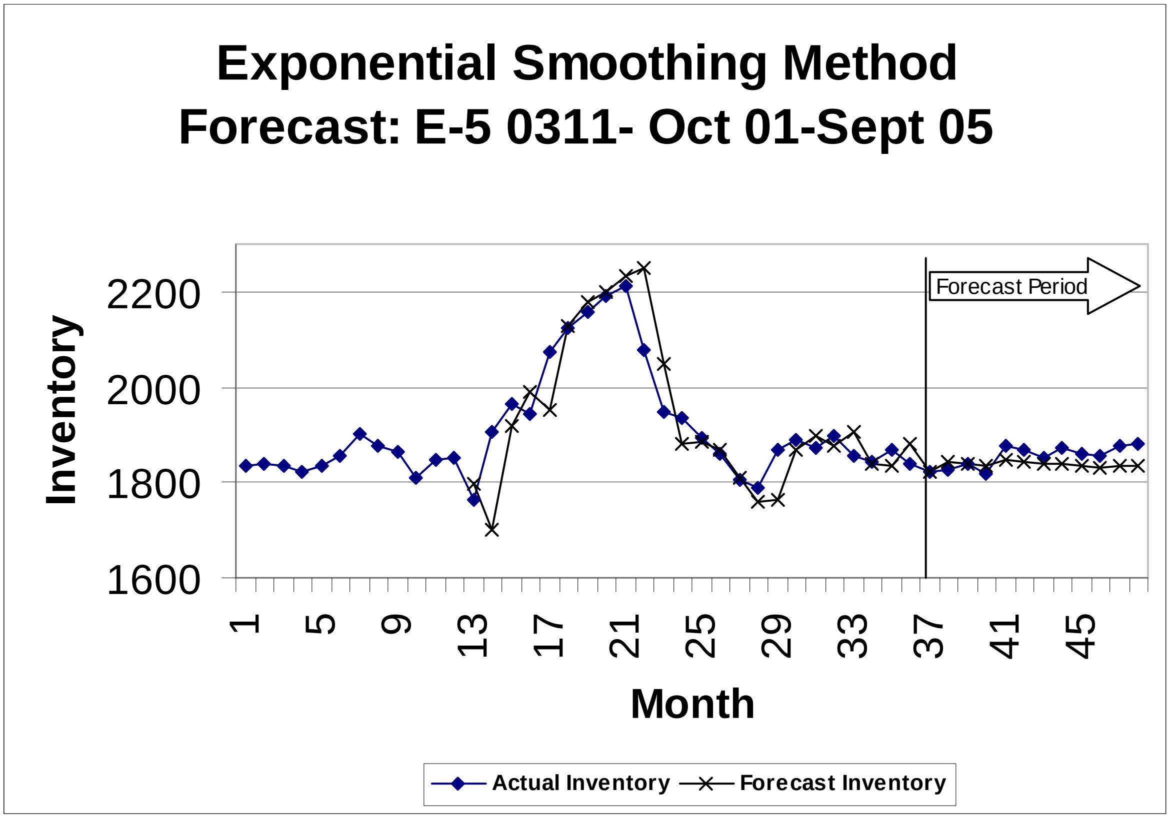

Figure 5. Exponential Smoothing Forecast Results: E-5 0311 – Oct 01-Sept 05

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VI. TIME SERIES DECOMPOSITION FORECASTING

A. OVERVIEW

Time series decomposition is a forecasting and analysis method frequently employed by the U.S. Bureaus of Census and Labor Statistics. Frederick R. Macaulay pioneered this technique in the 1920's at the National Bureau of Economic Research (Yaffee, 2000). The purpose of the research was the seasonal adjustment of time series data for analysis; however, refinements to the work have yielded practical and effective techniques to analyze cyclical behavior and create univariate forecasts.

The fundamental assumption of the decomposition technique is that every time series consists of a number of component parts that are related in some manner. As introduced in Chapter II, these components are trend (*T*), cycle (*C*), seasonality (*S*), and irregular (*I*) or random variation. The decomposition methodology views a time series as the sum or product of these attributes:[2]

$$x = T * S * C * I \tag{6.1}$$

This chapter examines time series decomposition forecasting techniques. Using the E-5 0311 Marine Corps enlisted population group time series introduced in Chapter IV, the forecasting model is developed over a multiple-staged process. The first section describes the model's process of smoothing random variation with moving average calculations. The following sections demonstrate the procedures for computing seasonality and estimating the trend. The chapter concludes by developing the forecast model, and creating the E-5 0311 manpower inventory forecast.

B. MOVING AVERAGES

Time series decomposition first requires the analyst to smooth the irregular component by calculating the series's moving averages. The forecaster is afforded some flexibility in choice of moving average techniques. For time series with higher levels of irregularity, Yaffee (2000) recommends longer period moving average calculations. Quarterly data should be smoothed with a five-period moving average (Shiskin, Young,

---

[2] Decomposition can be either a multiplicative or additive model. The multiplicative model is more commonly applied (Yaffee, 2000). This thesis describes the multiplicative forecasting method. In the additive model, multiplication is replaced by addition and division by subtraction.

& Musgrave, 1965). By convention, these are centered moving averages and may be weighted or unweighted depending on the analyst's preference (Stanford, 2002). The problem of loss of data at the end of a centered moving average is mitigated by allowing the number of elements in the set to diminish to the number of observations remaining as the end of the series approaches. This study developed and evaluated numerous combinations of moving average period length and weight. The five-month, centered moving average described in Equation (6.2) provides the best fit for these particular manpower inventory forecasting problems. Equations (6.3) and (6.4) describe how this study's model handles the centered moving average's loss of data for the final two observations in the series. Table 7 provides the moving average calculations for the E-5 0311 forecasting problem.

$$MA_i = \frac{\sum(x_{i-2} + x_{i-1} + x_i + x_{i+1} + x_{i+2})}{5} \tag{6.2}$$

$$MA_{t-1} = \frac{\sum(x_{t-3} + x_{t-2} + x_{t-1} + x_t)}{4} \tag{6.3}$$

$$MA_t = \frac{\sum(x_{t-2} + x_{t-1} + x_t)}{3} \tag{6.4}$$

Period	*x*	MA	Period	*x*	MA	Period	*x*	MA
1	1835		13	1765	1867.2	25	1895	1888.4
2	1840		14	1908	1886	26	1859	1857
3	1833	1832.8	15	1964	1930.8	27	1807	1843.8
4	1823	1837.2	16	1943	2003	28	1790	1842.6
5	1833	1849.6	17	2074	2052.6	29	1868	1845.4
6	1857	1858.4	18	2126	2098	30	1889	1863.6
7	1902	1867	19	2156	2152.2	31	1873	1876.8
8	1877	1862.2	20	2191	2152.6	32	1898	1871.6
9	1866	1860.6	21	2214	2116.8	33	1856	1867.6
10	1809	1850.2	22	2076	2072.4	34	1842	1860.4
11	1849	1827.8	23	1947	2013.2	35	1869	1851
12	1850	1836.2	24	1934	1942.2	36	1837	1849.333

Table 7. Moving Average Calculations; $\{x_1, x_2, \ldots, x_{36}\}$

This process smoothes the series of the irregular and seasonal components. Thus, the values produced by the moving average can be considered a product of each observation's trend and cyclical component (Shiskin *et al.*, 1965). Therefore, Equation (6.1) is modified in the following manner:

$$
\begin{aligned}
MA &= T * C \\
\therefore x &= MA * S * I \qquad (6.5) \\
\therefore \frac{x}{MA} &= S * I
\end{aligned}
$$

The ratio x/MA is referred to as the actual-to-moving-average ratio. This calculation allows the analyst to isolate the series's seasonal and irregular components. Table 8 depicts the actual-to-moving average ratio calculations for the forecasting problem example.

Period	*x* /MA	Period	*x* /MA	Period	*x* /MA
1		13	0.945266	25	1.003495
2		14	1.011665	26	1.001077
3	1.000109	15	1.017195	27	0.980041
4	0.992271	16	0.970045	28	0.971453
5	0.991025	17	1.010426	29	1.012247
6	0.999247	18	1.013346	30	1.01363
7	1.018747	19	1.001766	31	0.997975
8	1.007948	20	1.017839	32	1.014106
9	1.002902	21	1.045918	33	0.993789
10	0.977732	22	1.001737	34	0.99011
11	1.011599	23	0.967117	35	1.009724
12	1.007516	24	0.995778	36	0.993331

Table 8. Actual-to-Moving Average (x/MA) Calculations; $\{x_1, x_2, \ldots, x_{36}\}$

C. SEASONALITY

The analyst should now isolate the series's seasonality component, *S*. This is accomplished via a two-step process. As depicted in Table 9 below, the estimated seasonal index for each month is calculated by first averaging all the ratios for that particular month across all years of the observation set, and then, if required, renormalizing the ratios so that they sum to exactly the number of periods in a season. As the total of the monthly averages in Table 9 do not sum to twelve, the indices are adjusted in this particular example by multiplying each by the ratio of 12.00000/11.98964. The sum of the indices should correspond to the number of periods in a year; e.g., 52 for time series with weekly observations, twelve for monthly observations, and four for quarterly observations. The right-hand column of Table 9

depicts the time series's isolated seasonal component, *S*, which represents the percentage of normal typically observed in a particular month or season.

	FY 2002	FY 2003	FY 2003	Mean	Adjusted Index
Oct		0.945266	1.003495	0.97438	0.975222
Nov		1.011665	1.001077	1.006371	1.007241
Dec	1.000109	1.017195	0.980041	0.999115	0.999978
Jan	0.992271	0.970045	0.971453	0.977923	0.978768
Feb	0.991025	1.010426	1.012247	1.004566	1.005434
Mar	0.999247	1.013346	1.01363	1.008741	1.009612
Apr	1.018747	1.001766	0.997975	1.006163	1.007032
May	1.007948	1.017839	1.014106	1.013297	1.014173
Jun	1.002902	1.045918	0.993789	1.014203	1.015079
Jul	0.977732	1.001737	0.99011	0.98986	0.990715
Aug	1.011599	0.967117	1.009724	0.996147	0.997007
Sept	1.007516	0.995778	0.993331	0.998875	0.999738
Sum				11.98964	12

Table 9. Time Series Decomposition E-5 0311 Seasonal Index Calculations

The forecaster can now de-season the data. This is accomplished by dividing each value of the time series by the seasonal index, *S*. Thus, dividing Equation (6.1) by *S* yields:

$$\frac{x}{S} = T * C * I \tag{6.6}$$

The seasonally adjusted data, *x/S*, is depicted in the right-hand column of Table 10 below. In this example, the adjusted seasonal indices, *S*, are all very close in value to 1.0 suggesting the seasonality component has little effect on this series.

D. TREND

After calculating and isolating the time series's seasonality, the analyst can now estimate the trend component, *T*. This is accomplished by regressing the de-seasoned inventory level on the period number. Equation (6.7) provides the linear regression model calculated for the E-5 0311 forecasting problem. Table 11 depicts the trend calculation, *T*, for each observation.

$$T = 1895.075 + 0.7873 * period \tag{6.7}$$

Observation	Inventory *x*	Moving Average MA	*x* /MA	Adjusted Index S	Deseasoned Data *x* /S
1	1835			0.975222	1882
2	1840			1.007241	1827
3	1833	1832.8	1.000109	0.999978	1833
4	1823	1837.2	0.992271	0.978768	1863
5	1833	1849.6	0.991025	1.005434	1823
6	1857	1858.4	0.999247	1.009612	1839
7	1902	1867	1.018747	1.007032	1889
8	1877	1862.2	1.007948	1.014173	1851
9	1866	1860.6	1.002902	1.015079	1838
10	1809	1850.2	0.977732	0.990715	1826
11	1849	1827.8	1.011599	0.997007	1855
12	1850	1836.2	1.007516	0.999738	1850
13	1765	1867.2	0.945266	0.975222	1810
14	1908	1886	1.011665	1.007241	1894
15	1964	1930.8	1.017195	0.999978	1964
16	1943	2003	0.970045	0.978768	1985
17	2074	2052.6	1.010426	1.005434	2063
18	2126	2098	1.013346	1.009612	2106
19	2156	2152.2	1.001766	1.007032	2141
20	2191	2152.6	1.017839	1.014173	2160
21	2214	2116.8	1.045918	1.015079	2181
22	2076	2072.4	1.001737	0.990715	2095
23	1947	2013.2	0.967117	0.997007	1953
24	1934	1942.2	0.995778	0.999738	1935
25	1895	1888.4	1.003495	0.975222	1943
26	1859	1857	1.001077	1.007241	1846
27	1807	1843.8	0.980041	0.999978	1807
28	1790	1842.6	0.971453	0.978768	1829
29	1868	1845.4	1.012247	1.005434	1858
30	1889	1863.6	1.01363	1.009612	1871
31	1873	1876.8	0.997975	1.007032	1860
32	1898	1871.6	1.014106	1.014173	1871
33	1856	1867.6	0.993789	1.015079	1828
34	1842	1860.4	0.99011	0.990715	1859
35	1869	1851	1.009724	0.997007	1875
36	1837	1849.333	0.993331	0.999738	1837

Table 10. De-seasoned E-5 0311 Inventory Calculations

Observation	Inventory *x*	Moving Average MA	*x* /MA	Adjusted Index S	Deseasoned Data *x* /S	Least Sq Estimation T
1	1835			0.975222	1882	1895.8622
2	1840			1.007241	1827	1896.6495
3	1833	1832.8	1.000109	0.999978	1833	1897.4368
4	1823	1837.2	0.992271	0.978768	1863	1898.224
5	1833	1849.6	0.991025	1.005434	1823	1899.0113
6	1857	1858.4	0.999247	1.009612	1839	1899.7985
7	1902	1867	1.018747	1.007032	1889	1900.5858
8	1877	1862.2	1.007948	1.014173	1851	1901.3731
9	1866	1860.6	1.002902	1.015079	1838	1902.1603
10	1809	1850.2	0.977732	0.990715	1826	1902.9476
11	1849	1827.8	1.011599	0.997007	1855	1903.7348
12	1850	1836.2	1.007516	0.999738	1850	1904.5221
13	1765	1867.2	0.945266	0.975222	1810	1905.3094
14	1908	1886	1.011665	1.007241	1894	1906.0966
15	1964	1930.8	1.017195	0.999978	1964	1906.8839
16	1943	2003	0.970045	0.978768	1985	1907.6712
17	2074	2052.6	1.010426	1.005434	2063	1908.4584
18	2126	2098	1.013346	1.009612	2106	1909.2457
19	2156	2152.2	1.001766	1.007032	2141	1910.0329
20	2191	2152.6	1.017839	1.014173	2160	1910.8202
21	2214	2116.8	1.045918	1.015079	2181	1911.6075
22	2076	2072.4	1.001737	0.990715	2095	1912.3947
23	1947	2013.2	0.967117	0.997007	1953	1913.182
24	1934	1942.2	0.995778	0.999738	1935	1913.9692
25	1895	1888.4	1.003495	0.975222	1943	1914.7565
26	1859	1857	1.001077	1.007241	1846	1915.5438
27	1807	1843.8	0.980041	0.999978	1807	1916.331
28	1790	1842.6	0.971453	0.978768	1829	1917.1183
29	1868	1845.4	1.012247	1.005434	1858	1917.9056
30	1889	1863.6	1.01363	1.009612	1871	1918.6928
31	1873	1876.8	0.997975	1.007032	1860	1919.4801
32	1898	1871.6	1.014106	1.014173	1871	1920.2673
33	1856	1867.6	0.993789	1.015079	1828	1921.0546
34	1842	1860.4	0.99011	0.990715	1859	1921.8419
35	1869	1851	1.009724	0.997007	1875	1922.6291
36	1837	1849.333	0.993331	0.999738	1837	1923.4164

Table 11. Time Series Decomposition E-5 0311 Trend Calculations

E. MULTIPLICATIVE DECOMPOSITION FORECASTING

After computing the adjusted seasonal indices and trend calculations, the analyst can construct a forecasting model. Equation (6.8) provides the general multiplicative decomposition-forecasting model, while Equation (6.9) details the specific model applicable to the E-5 0311 forecasting problem.

$$\hat{x}_i = S_i * T_i \tag{6.8}$$

$$\hat{x}_i = S_i * (1895.075 + 0.7873i) \tag{6.9}$$

Table 12 presents the predicted inventory level for each forecast period, the observed (actual) inventory level, and the absolute percentage error. The absolute percentage error, described in Equation (5.25), is one of many possible measures of forecast effectiveness. Figure 7 provides a graphical presentation of the observed and forecast inventory levels for each period.

Multiplicative Decomposition Forecast Results			
Forecast Period	Actual Inventory	Predicted Inventory	Absolute Percentage Error
6-Month	1869	1947	4.17%
12-Month	1880	1932	2.77%

Table 12. Multiplicative Decomposition Forecast Results

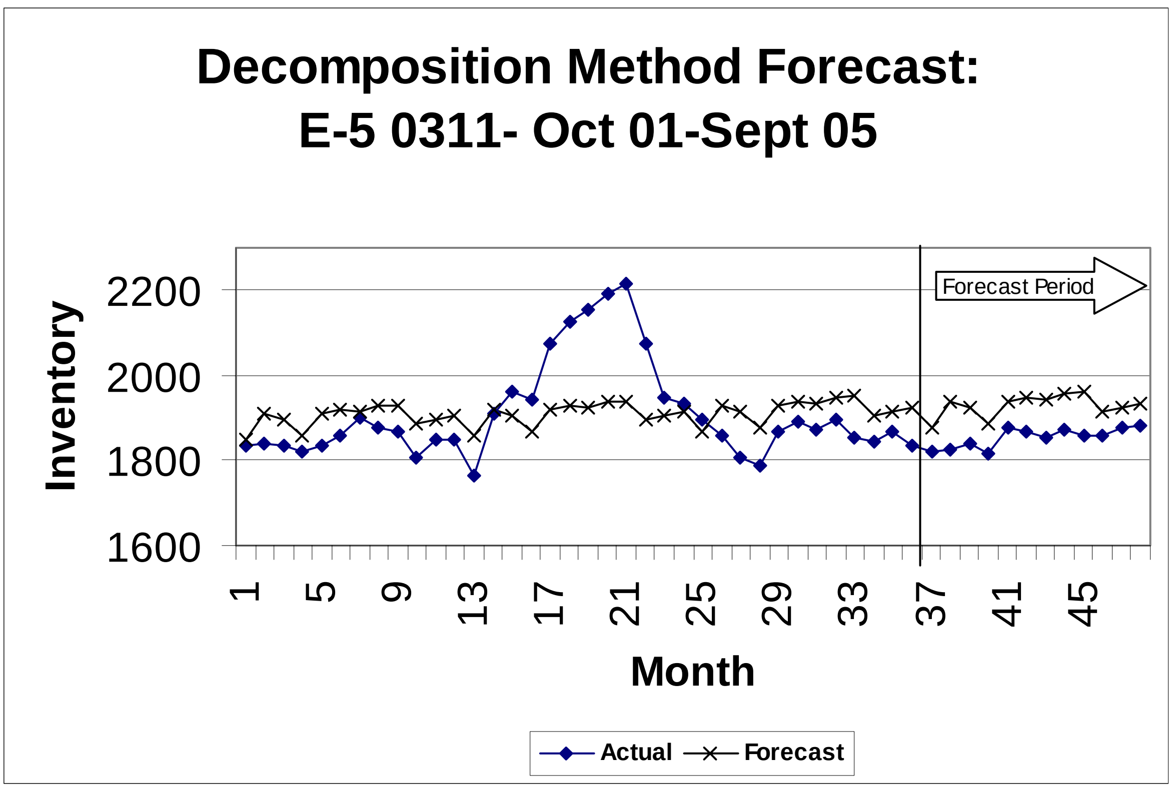

Figure 6. Multiplicative Decomposition Forecast Results: E-5 0311: Oct 01-Sept 05

F. CHAPTER SUMMARY

Time series decomposition forecasting is based on the concept that underlying factors of a time series can be identified and isolated. These components are trend, cycle, seasonality, and irregular occurrences. The analyst develops a forecast model by first identifying and removing the component effects from the data (decomposition). After the effects are identified and isolated, the analyst creates a prediction by reassembling the components (recomposition). This technique is a common and effective method of forecasting time series behavior.

VII. BOX-JENKINS FORECASTING

A. OVERVIEW

In 1970, George Box and Gwilym Jenkins devised a methodology for analyzing and forecasting univariate time series data (Hoff, 1983). Unlike the automatic forecasting approaches detailed in Chapters V and VI, the Box-Jenkins method is not a forecasting algorithm per se. It is a "strategy" for identifying and selecting a forecasting technique from a general class of statistical models (Chatfield, 1996). This approach creates forecasts by combining two common time series modeling methods, the autoregression (AR) and moving average (MA) techniques. The Box-Jenkins process assumes that any given time series observation can be modeled as a function of its past values (derived through autoregression) and the current and past values of random errors or white noise (derived through a moving average).

This chapter's purpose is to introduce the concepts of selecting and developing a forecasting model with the Box-Jenkins techniques.[3] The first section discusses the data preparations required to ready the time series for analysis and modeling with the Box-Jenkins approach. The following section examines procedures for autoregressive and moving average model selection. Next, the chapter reviews the model evaluation process. The chapter concludes by discussing the Box-Jenkins forecast created for the E-5 0311 Marine Corps enlisted population group introduced in Chapter IV.

B. DATA PREPARATION

The Box-Jenkins procedure makes three fundamental assumptions regarding the data's characteristics. First, the procedure requires that the time series be discrete and the observations equally spaced over time (Yaffee, 2000). Second, the data must have no missing observations. Yaffee (2000) discusses several options for developing algorithms to replace missing values if required. Finally, the data must be rendered stationary in the mean and variance (Chatfield, 1996).

[3] The scope of this thesis limits the discussion of the Box-Jenkins approach to time series analysis and forecasting to a brief overview. For a detailed description of the Box-Jenkins approach to time series analysis and forecasting, see Yaffee (2000) and Gujarati (2003).

Data preparation entails transformations and differencing. Transformations, such as logarithms and square roots, can stabilize variance in an observation set where the variation changes with the level. Differencing involves manipulating the time series until there are no discernable indications of trend or seasonality. This procedure requires taking the difference between sequential observations. Most time series are nonstationary and need some differencing and/or transformation prior to Box-Jenkins analysis and modeling. The E-5 0311 monthly inventory level time series requires logarithmic transformation and first order differencing in order to achieve stationarity.

1. Transformation– Stabilizing the Variance

A stationary time series has a constant underlying variance. Often, time series data will demonstrate fluctuating variance. Graphing the observations will frequently expose this volatility. According to Yaffee (2000), "If the variation in the series expands, contracts, or fluctuates with the passage of time, the change in variation will usually be apparent in a time plot."

If instability in variance is present, the forecaster can affect stationarity via a number of possible transformations. A power, logarithmic or Box-Cox transformation may stabilize the variance. Power transformations include the square, square root, cube, and cube root of the original series.

A logarithmic transformation effectively stabilizes the variance in the mean of the E-5 0311 population group observation set. Table 13 lists the log transformed time series observations.

2. Differencing – Stabilizing the Mean

Box-Jenkins analysis and modeling requires not only variance stationarity, but mean stationarity as well (Yaffee, 2000). This implies no trend or seasonality. Deviations about the mean are temporary and the observations demonstrate equilibrium about the mean over the term of the series (Sherry, 1984). The analyst should conduct a preliminary analysis of the data by simply sketching a time plot and inspecting it for any observable patterns in the data. Any clear upward or downward trend or seasonality would indicate that the data should be differenced prior to continuing the Box-Jenkins analysis.

First order differencing is accomplished with the equation:

$$Z_i = x_i - x_{i-1}, i = 2,...,t \tag{7.1}$$

where Z_i replaces x_i in the differenced series. The difference process should be repeated as required to achieve stationarity.

Correlograms are also a useful diagnostic tool for determining the need for differencing. Rapid attenuation of the plotted autocorrelation function suggests the series is sufficiently differenced (Chatfield, 1996). If the autocorrelations increase, decay slowly, exhibit a wave-like cyclical pattern, or decrease linearly, passing through zero to become negative, the series is not stationary and should be differenced one or more times (Yaffee, 2000). Figure 8 depicts a correlogram of the E-5 0311 time series introduced in Chapter IV. The decreasing linear pattern depicted by the shaded graph indicates the series is not stationary. Figure 9 depicts the series after first order differencing and reflects a series that is sufficiently stationary in the mean to continue the Box-Jenkins analysis. The analyst can employ the Dickey-Fuller test if a more sophisticated test for stationarity is required (Yaffee, 2000). Table 13 provides the log transformed, first differenced E-5 0311 inventory time series observations.

Period	x	log(x)	$\log(x_i)-\log(x_{i-1})$	Period	x	log(x)	$\log(x_i)-\log(x_{i-1})$
1	1835	3.2636		19	2156	3.3336	0.0061
2	1840	3.2648	0.0012	20	2191	3.3406	0.0070
3	1833	3.2632	-0.0017	21	2214	3.3452	0.0045
4	1823	3.2608	-0.0024	22	2076	3.3172	-0.0280
5	1833	3.2632	0.0024	23	1947	3.2894	-0.0279
6	1857	3.2688	0.0056	24	1934	3.2865	-0.0029
7	1902	3.2792	0.0104	25	1895	3.2776	-0.0088
8	1877	3.2735	-0.0057	26	1859	3.2693	-0.0083
9	1866	3.2709	-0.0026	27	1807	3.2570	-0.0123
10	1809	3.2574	-0.0135	28	1790	3.2529	-0.0041
11	1849	3.2669	0.0095	29	1868	3.2714	0.0185
12	1850	3.2672	0.0002	30	1889	3.2762	0.0049
13	1765	3.2467	-0.0204	31	1873	3.2725	-0.0037
14	1908	3.2806	0.0338	32	1898	3.2783	0.0058
15	1964	3.2931	0.0126	33	1856	3.2686	-0.0097
16	1943	3.2885	-0.0047	34	1842	3.2653	-0.0033
17	2074	3.3168	0.0283	35	1869	3.2716	0.0063
18	2126	3.3276	0.0108	36	1837	3.2641	-0.0075

Table 13. The log transformed, first differenced E-5 0311 inventory time series

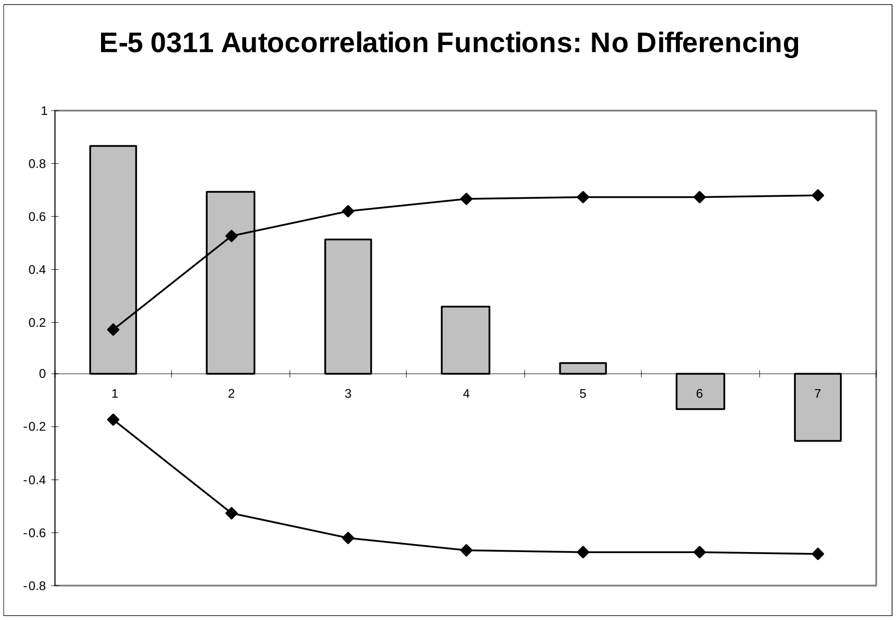

Figure 7. E-5 0311 Autocorrelation Functions: No Differencing

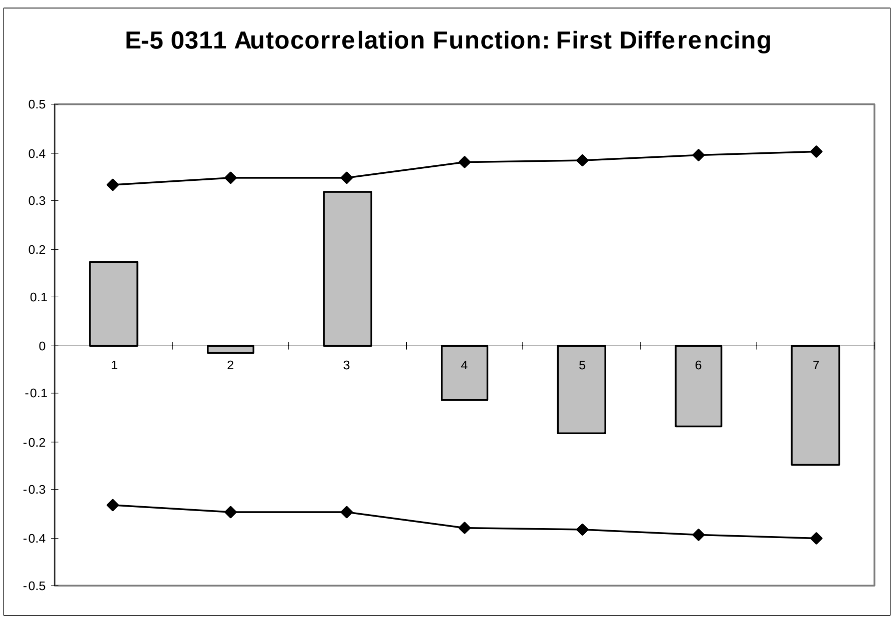

Figure 8. E-5 0311 Autocorrelation Functions: First Differencing

C. MODEL SELECTION

The Box-Jenkins approach provides a methodology for selecting, developing, and evaluating forecasting models from a broad class of time series processes. These include autoregressive (AR) and moving average (MA) techniques. AR models assume that the current value of a series is dependent on previous observations with some random error

component (Chatfield, 1996). MA models assume the current value of the series is dependent on external influences. A combination of these techniques is called an autoregressive moving average model (ARMA). This method isolates and models both the time series's underlying process and external influences. This systematic approach and the large class of available techniques empower the Box-Jenkins procedure to effectively model a wide spectrum of time series behavior.

1. Autoregressive Models

The analyst should consider the AR process when it is plausible to assume that the current observation of the series is dependent on recent previous values with some additional random error (Chatfield, 1996). The Marine Corps enlisted inventory time series discussed in this thesis are examples of data that fit this description. With the manpower inventory process, the current population level is dependent on the previous level. The error term accounts for promotions, discharges, and other losses and gains.

The error term is subject to several assumptions. It must have a Normal distribution, a mean of zero, and a constant variance in order for AR modeling to be indicated (Yaffee, 2000). An autoregressive process of order *p* [AR(*p*)], is of the form

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \ldots + \phi_p Z_{t-p} + \varepsilon_t \tag{7.2}$$

where ϕ_i denotes the autoregressive coefficients and ε is the error term. An AR (1) model is defined by Equation (7.3)

$$Z_t = \phi Z_{t-1} + \varepsilon_t . \tag{7.3}$$

If the model does not satisfy Equation (7.4) below, the series does not meet stationarity requirement and requires additional differencing or transformation (Yaffee, 2000).

$$|\phi| < 1 \tag{7.4}$$

The autocorrelation function (ACF) and partial autocorrelation function (PACF) are helpful tools for AR model selection.[4] The construction of correlograms simplifies the interpretation of ACF and PACF parameters. Table 14 details the AR model selection criteria given an ACF and PACF correlogram for a given time series. Figures 10 and 11 illustrate the autocorrelation and partial autocorrelation behavior of AR (1) and AR (2) models, respectively.

AR Model Selection Criteria
1. If none of the simple autocorrelations is significantly different from zero, the series is essentially a random number or white-noise series, which is not amenable to autoregressive modeling.
2. If the simple autocorrelations decrease linearly, passing through zero to become negative, or if the simple autocorrelations exhibit a wave-like cyclical pattern, passing through zero several times, the series is not stationary; it must be differenced one or more times before it may be modeled with an autoregressive process.
3. If the simple autocorrelations exhibit seasonality; i.e., there are autocorrelation peaks every dozen or so (in monthly data) lags, the series is not stationary; it must be differenced with a gap approximately equal to the seasonal interval before further modeling.
4. If the simple autocorrelations decrease exponentially but approach zero gradually, while the partial autocorrelations are significantly non-zero through some small number of lags beyond which they are not significantly different from zero, the series should be modeled with an autoregressive process.
5. If the partial autocorrelations decrease exponentially but approach zero gradually, while the simple autocorrelations are significantly non-zero through some small number of lags beyond which they are not significantly different from zero, the series should be modeled with a moving average process.
6. If the partial and simple autocorrelations both converge upon zero for successively longer lags, but neither actually reaches zero after any particular lag, the series may be modeled by a combination of autoregressive and moving average process.

Table 14. Autoregressive Model Selection Criteria (From: Arsham, 1994)

[4] "The partial autocorrelation measures correlation between time series observations that are *k* time periods apart after controlling for correlations at intermediate lags (i.e. lags less than *k*). Partial autocorrelation is the correlation between Z_t and Z_{t-k} after removing the effects of the intermediate Z's." (Gujarati, 2003)

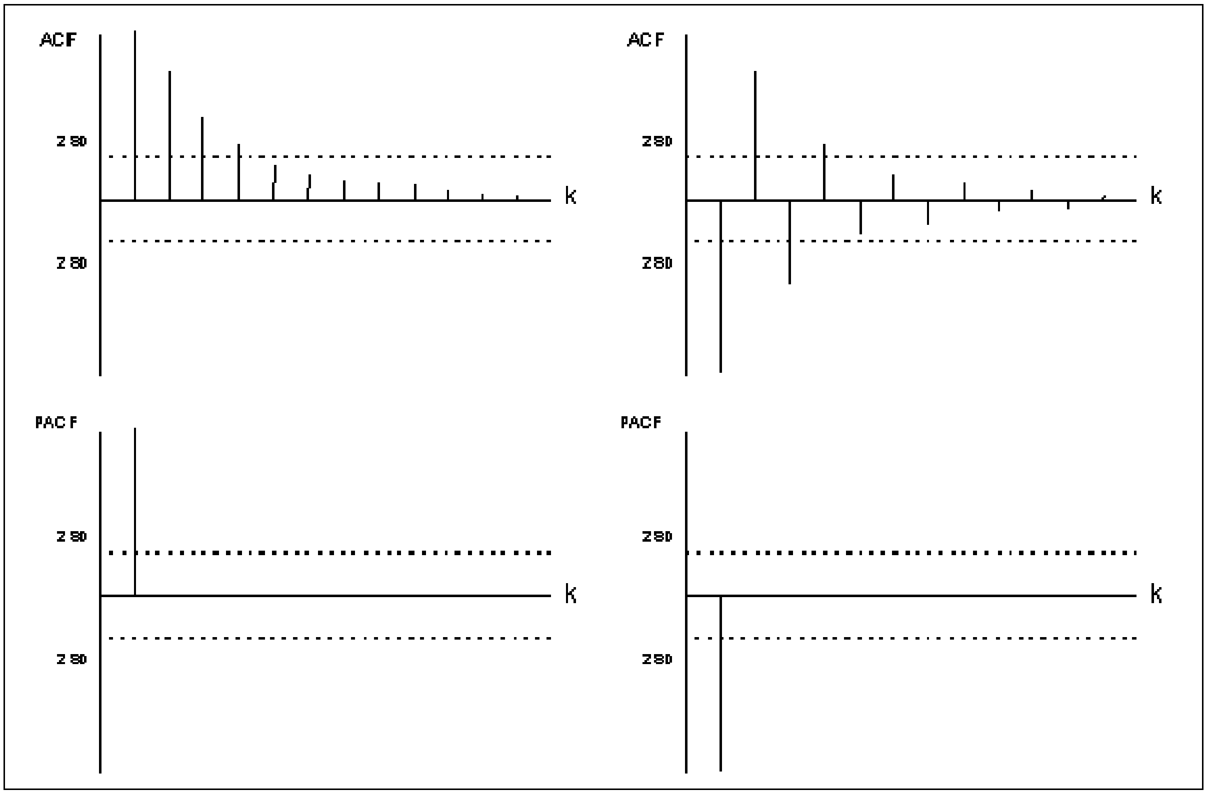

Figure 9. AR (1) Model ACF and PACF Behavior (From: Borhers & Wessa, 2000)

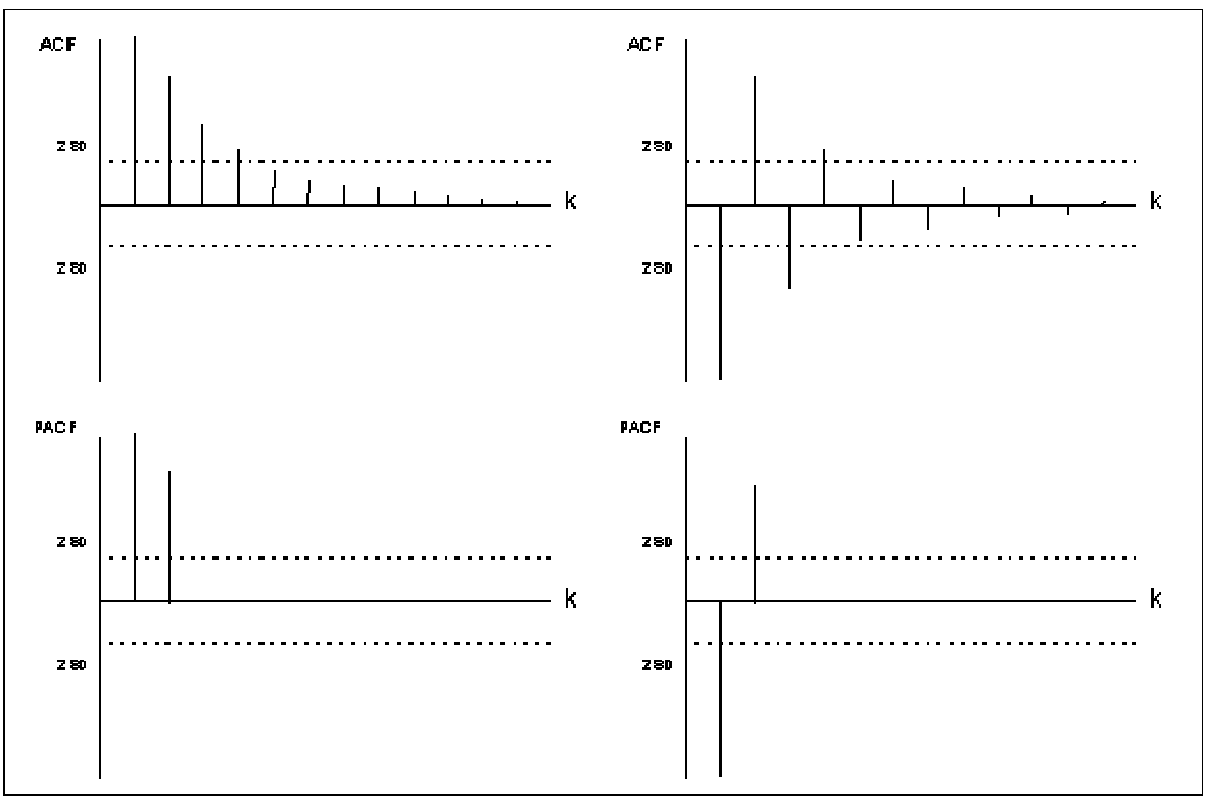

Figure 10. AR (2) Model ACF and PACF Behavior (From: Borhers & Wessa, 2000)

2. Moving Average Models

A time series is influenced by a moving average process if the current observation can be expressed as a linear function of the current error and one or more previous error terms (Sherry, 1984). This process is one affected by a variety of "random," external influences. These events have an immediate effect and may also affect observation values, to a certain extent, for several subsequent periods (Chatfield, 1996). Examples would be the effects of strikes or natural disasters on a time series of economic indicators.

A moving average process of order *q* [MA(*q*)]is expressed:

$$\hat{Z}_t = \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q} \tag{7.5}$$

where θ_i denotes the moving average parameters and ε_i is the random error term. This error term is subject to the same assumptions as the AR error term. Specifically, it must be normally distributed, have a mean of zero, and have a constant variance. Equation (7.6) expresses a MA (1) model.

$$\hat{Z}_t = \varepsilon_t - \theta \varepsilon_{t-1} \tag{7.6}$$

As with the AR model identification and selection process, correlograms plotting the ACF and PACF are useful for MA model selection. If the ACF of the differenced series displays an abrupt drop to zero while the PACF decays more slowly, the series is likely influenced by a MA process. Examination of the ACF plot reveals the likely order, *q*, of the moving average model. If the autocorrelation is significant at lag *k*, but not at any higher lags (the ACF drops to zero at lag *k+1*), a moving average model of order *k* [MA (*k)*] is indicated. Figure 12 graphically depicts the behavior of the ACF and PACF in MA (1) models while Figure 13 depicts the behavior of those functions in MA (2) models.

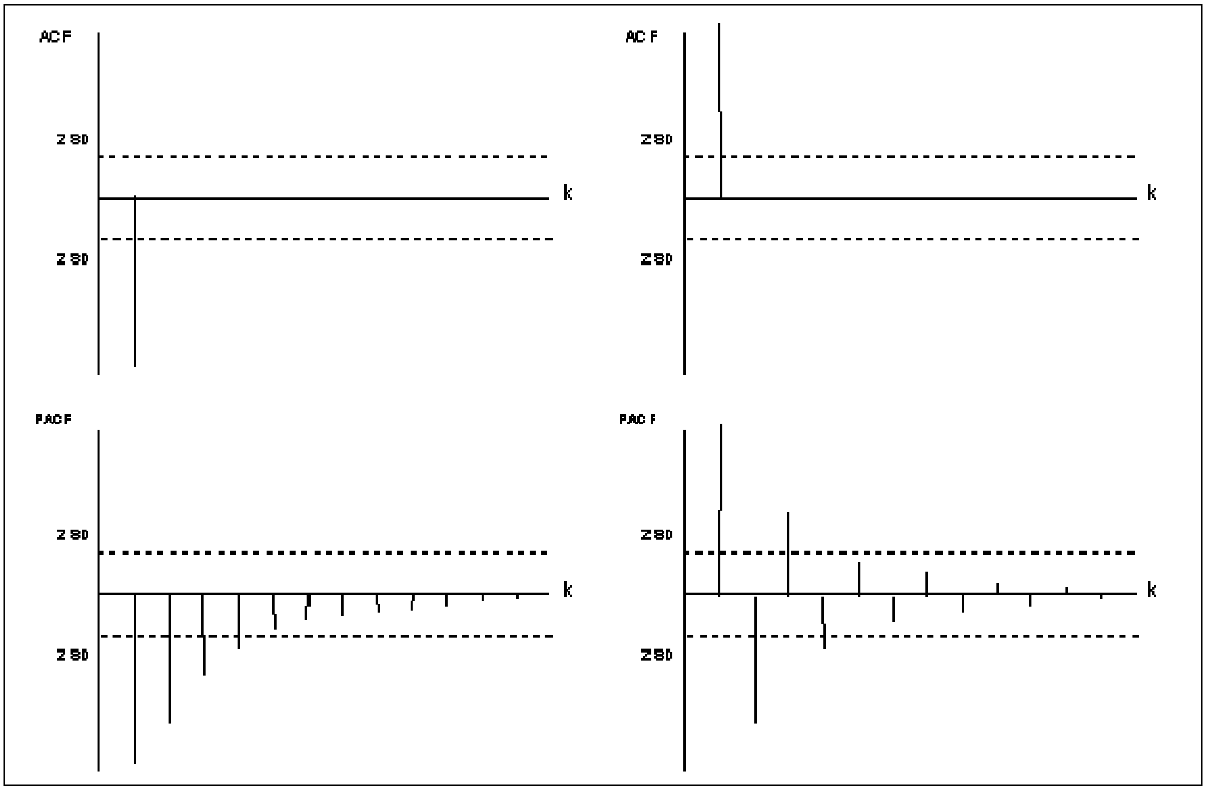

Figure 11. MA (1) Model ACF and PACF Behavior (From: Borhers & Wessa, 2000)

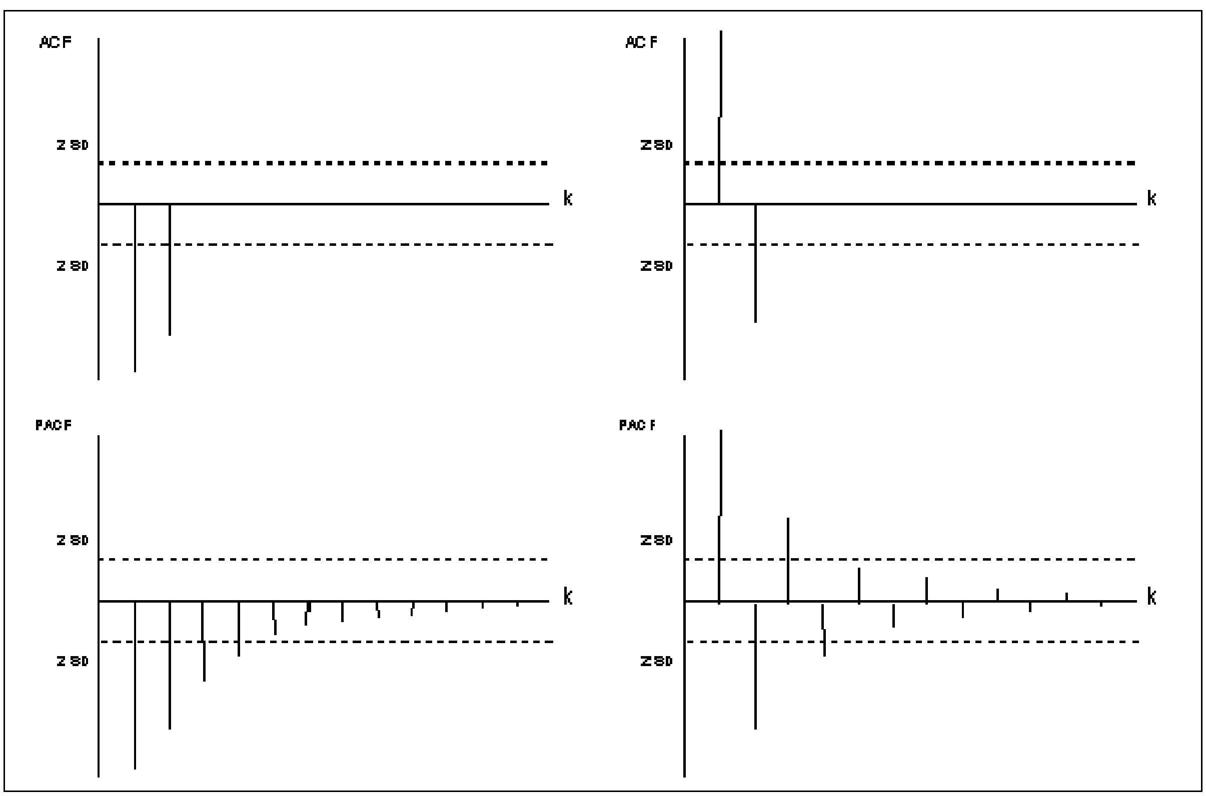

Figure 12. MA (2) Model ACF and PACF Behavior (From: Borhers & Wessa, 2000)

3. Autoregressive Moving Average Models

Autoregressive moving average (ARMA) models integrate the AR and MA processes. These models express the relationship of the current observation as a linear function of past observations and error terms. As previously noted, an autoregressive process of order *p* is typically denoted as AR (*p*). A moving average process with *q* lag terms is conventionally classified as MA (*q*). A combination model containing *p* AR terms and *q* moving average terms is classified as ARMA (*p,q).*

If the time series is differenced *d* times to meet the constraints of stationarity, the model is classified as a ARIMA (*p*,*d*,*q*) model. The "I" denotes that the model is "integrated." The equation for this model is:

$$\hat{Z}_t = \theta_0 + \phi_1 Z_{t-1} + \ldots + \phi_p Z_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \ldots - \theta_q \varepsilon_{t-q} \tag{7.7}$$

The autocorrelation and partial autocorrelation functions are used to select model candidates as described in the previous sections.

D. MODEL EVALUATION

The next step in the Box-Jenkins process is to estimate the parameters and evaluate one or more models. Parameter estimation involves calculating the model coefficients that provide the best fit for the data and is frequently accomplished with statistical software. Once the models' parameters have been estimated, the analyst must evaluate them for accuracy assessing the autocorrelation function of the residuals, the *t*-ratios, and computing the minimum sum of squares or some other goodness-of-fit indicator for comparison purposes (Sherry, 1984).

The autocorrelation function of each model's residuals should be random about the mean and the mean should be zero. Additionally, the ACF should have a constant variance and a magnitude of less than two standard errors. The t-ratio is calculated for each parameter estimate by the series's standard deviation. This ratio should be greater than +/- 2.0 indicating that the coefficient is significantly different from zero. The sum of squares measurement is useful for comparing the goodness-of-fit between two or more models.

Often more than one model will satisfy the evaluation criteria. Under these circumstances, the analyst should apply the principle of parsimony and select the lowest-order model available that satisfies the analyst's forecast criteria.[5]

E. FORECASTING

A Box-Jenkins analysis of the E-5 0311 time series data indicates that a log-transformed ARIMA (1,1,1) model is appropriate for this particular population group. Modifying Equation (7.7), an ARIMA (1,1,1) model can be expressed mathematically as:

$$Z_t = \theta_0 + \phi_1 Z_{t-1} + \varepsilon_t - \theta_1 \varepsilon_{t-1} \tag{7.8}$$

Recalling Equation (7.1), this formula is rewritten to the following form for models with first order differencing (O'Donovon, 1983):

$$\begin{aligned} x_t - x_{t-1} &= \theta_0 + \phi_1 (x_{t-1} - x_{t-2}) + \varepsilon_t - \theta_1 \varepsilon_{t-1} \\ \therefore x_t &= \theta_0 + (1+\phi_1) x_{t-1} - \phi_1 x_{t-2} + \varepsilon_t - \theta_1 \varepsilon_{t-1} \end{aligned} \tag{7.9}$$

Table 15 details the model's parameter estimates calculated by the SAS Time Series Forecasting System software application.

Model Parameter	Estimate	Std Error	t	Prob>|t|
Intercept	-0.0003106	0.0058	-0.0534	0.9577
MA Lag (1)	-0.94190	0.386	-6.7975	<.0001
AR Lag (1)	-0.5296	0.1927	2.2472	0.0980
Model Variance	0.0007624			

Table 15. E-5 0311 Log ARIMA (1,1,1) Parameter Estimates

Substituting the ARIMA process coefficients outlined in Table 15 above, the analyst can mathematically express the E-5 0311 inventory data as:[6]

$$\hat{Z}_t = -0.0003106 - 0.5296 * Z_{t-1} + \varepsilon_t + 0.94190 \varepsilon_{t-1} \tag{7.10}$$

Table 16 presents the predicted inventory level for each forecast period, the observed (actual) inventory level, and the absolute percentage error. The absolute percentage error, described in Equation (5.25), is one of many possible measures of forecast effectiveness. Figure 14 provides a graphical presentation of the observed and forecast inventory levels for each period.

[5] Chapter II discusses forecast criteria.

[6] This model requires a log to linear conversion to complete the inventory forecast computations.

Log ARIMA (1,1,1) Forecast Results			
Forecast Period	Actual Inventory	Predicted Inventory	Absolute Percentage Error
6-Month	1869	1830	2.09%
12-Month	1880	1833	1.44%

Table 16. E-5 0311 Log ARIMA (1,1,1) Forecast Results

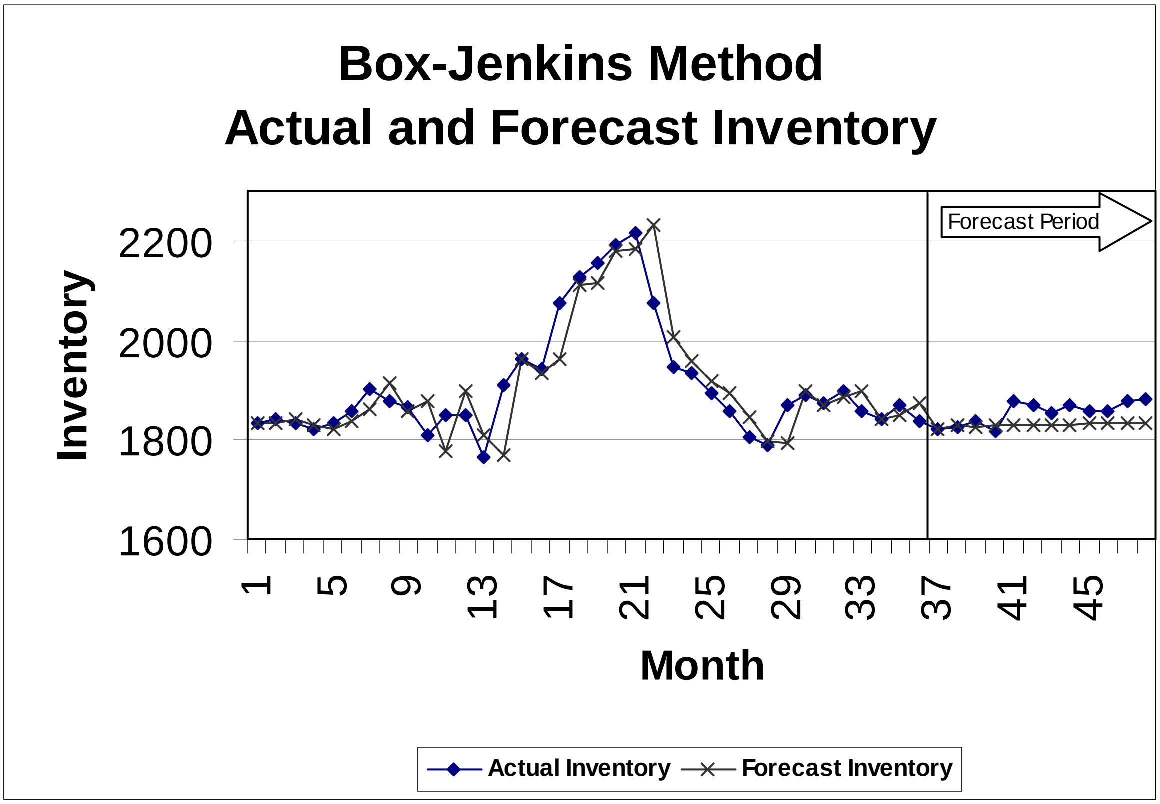

Figure 13. Log ARIMA (1,1,1) Forecast Results: E-5 0311 – Oct 01-Sept 05

The Box-Jenkins technique is not an automated forecasting method. Each time series model requires separate analysis, development and evaluation. The log ARIMA (1,1,1) model developed for the E-5 0311 time series is not suitable for every Marine Corps enlisted population group considered in this thesis. The Appendix details the Box-Jenkins forecast developed for each series.

Because the technique emphasizes the analysis and evaluation processes, it frequently yields more effective models than generically applying automated forecasting methods. However, the Box-Jenkins method is a complex approach that often requires a "large expenditure of time and effort" (Chatfield, 1996). Although statistical computer applications can facilitate the development of Box-Jenkins models, automatic techniques

are generally more suitable when there are “large numbers of series to be forecast” (Chatfield, 1996).

F. CHAPTER SUMMARY

The Box-Jenkins technique is a sophisticated approach by which to analyze time series data and extrapolate a forecast. This method provides a framework for identifying and selecting a model and creates forecasts by combining two common time series modeling methods, the autoregression and moving average techniques. These processes assume any given time series observation can be modeled as a function of its past values and the current and past values of random errors. The Box-Jenkins technique is appropriate for developing forecasts for a small number of series or when the analyst has a significant interest in diagnosing and analyzing the fundamental components affecting a particular time series.

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VIII. RESULTS

A. OVERVIEW

This chapter evaluates the six- and twelve-month predictive capability of the time series forecasting models developed in this thesis. The first section provides an overview of evaluation techniques and considerations. Several useful measures of effectiveness are introduced. The following sections compare the forecast results with respect to the Mean Absolute Percentage Error, the Sum of Squared Errors, and the Mean Absolute Error criteria respectively. The chapter concludes with an analysis of the results.

B. EVALUATING FORECASTS

Forecast evaluation is an essential part of the model development process. Methods for evaluating forecast models are based on conventional analytical procedures. The analyst should evaluate the models under situations that match the forecasting problem. One effective method is to back-forecast, employing older observations in the time series to develop the model, while comparing the forecast fit against the actual values of recent, out-of-sample observations (Wooldridge, 2003). Yaffee (2000), notes that a model's statistical accuracy should not be the single criterion employed when evaluating a particular forecasting technique. The analyst should also develop and consider some assessment of cost in terms of time, money, and effort required for each method.

When evaluating the effectiveness of forecast methods across several series, the analyst should utilize error measurements that consider the scale of the observations (Armstrong, 2001). This measurement must not be subject to distortion due to a disparity in series observation value magnitudes (thus weighting some samples more heavily). Some forecasts perform well when evaluated against one criterion yet are outperformed by other models when judged by other criteria (Yaffee, 2000). Consequently, the analyst should employ and consider multiple error measures when evaluating models (Armstrong, 2001).

This thesis develops manpower inventory forecasting models for 44 MOS/paygrade combinations. The Appendix summarizes the time series forecasts

constructed for each population group. Each observation set contains 48 monthly observations starting in October 2001 and concluding in September 2005. The estimation sub-sample consists of the first 36 observations and is used to estimate each model's parameters. The validation sub-sample consists of the final 12 observations and is reserved for evaluating each model's forecast accuracy with respect to actual inventory levels.

Three error measurements are offered to comparatively evaluate the models against each other and the current forecasting method.[7] These are the Mean Absolute Percentage Error (MAPE), Sum of Squared Errors (SSE), and Mean Absolute Error (MAE). Equations (8.1), (8.2), and (8.3) respectively describe these assessment-of-fit indicators.

$$MAPE = \frac{\sum_{i=1}^{T} \frac{\left|(x_i - \hat{x}_i)\right|}{x_i}}{T} * 100 \tag{8.1}$$

$$SSE = \sum_{i=1}^{T} (x_i - \hat{x}_i)^2 \tag{8.2}$$

$$MAE = \frac{\sum_{i=1}^{T} \left|(x_i - \hat{x}_i)\right|}{T} \tag{8.3}$$

where T = number of forecasts analyzed,
i = the forecast period,
x_i = the actual observed value at time i,
and $\hat{x}_i$ = the forecast value at time i.

C. RESULTS

1. Mean Absolute Percentage Error

The Mean Absolute Percentage Error is a general assessment of fit useful in comparing the effectiveness of different models (Yaffee, 2001). The scale of the time series observations influences this particular error measurement. Consequently, a forecast error of 1,000 in a population group of 10,000 Marines carries the identical

[7] The Enlisted Staffing Goal Model's inventory forecasting method, described in Chapter III, produces a single forecast — regardless of the forecast horizon. Consequently, in this evaluation, the current method's six- and twelve-month forecasts are the same.

weight as a forecast error of 10 in a population group of 100 Marines. Table 17 provides the six- and twelve-month MAPE comparisons for the forecasting techniques considered in this study. A lower value indicates a better forecast fit.

	Six-Month Forecast	Twelve-Month Forecast
Current Method	10.2%	10.7%
Exponential Smoothing	11.0%	12.7%
Decomposition	12.3%	13.2%
Box-Jenkins	10.5%	10.7%

Table 17. Forecast Model Comparison - Mean Absolute Percentage Error (MAPE)

2. Sum of Squared Errors

The Sum of Squared Errors is a common statistical method for evaluating and comparing a model's accuracy. This statistic provides the analyst a sense of the forecast's dispersion of error (Yaffee, 2000). Table 18 compares the respective models' six- and twelve-month forecast SSE calculations. A lower value indicates a superior fit.

	Six-Month Forecast	Twelve-Month Forecast
Current Method	1,529,234	1,236,988
Exponential Smoothing	475,752	300,874
Decomposition	196,944	159,051
Box-Jenkins	475,045	248,693

Table 18. Forecast Model Comparison - Sum of Squared Errors (SSE)

3. Mean Absolute Error

The Mean Absolute Error is a weighted average of the absolute errors, with the relative frequencies as the weight factors. Simply put, it represents each forecasting technique's average error amount. Table 19 compares the models' six- and twelve-month forecast SSE calculations. A lower value indicates a superior forecast fit.

	Six-Month Forecast	Twelve-Month Forecast
Current Method	47	49
Exponential Smoothing	34	37
Decomposition	28	30
Box-Jenkins	30	32

Table 19. Forecast Model Comparison – Mean Absolute Error (MAE)

4. E1-E4 Paygrade Forecast Evaluation

Table 20 lists the six- and twelve-month forecast error measurement calculations for a sub-set of the population group forecasts included in this study. This sub-set is comprised of the ten E1-E4 paygrade time series.

	6-Month Forecast			12-Month Forecast		
	MAPE	**SSE**	**MAE**	**MAPE**	**SSE**	**MAE**
Current Method	13.4%	1,508,894	38	15.9%	1,214,237	39
Exponential Smoothing	10.3%	455,648	23	17.2%	247,724	22
Decomposition	9.9%	176,113	16	21.5%	136,510	19
Box-Jenkins	10.8%	467,770	22	12.3%	225,494	21

Table 20. E1-E4 Paygrade Time Series Forecast Model Comparisons

D. ANALYSIS

On the whole, the accuracy of time series forecasts improves, as more observations are included (Yaffee, 2000). This axiom is dependant on the data being

relevant to the forecast (Yaffee, 2000). Under normal circumstances many of these forecast techniques would benefit from an additional year or two of monthly inventory level observations. However, these data sets are constrained to 36-months in length to better reflect the impact of policy changes since Operation IRAQI FREEDOM and the Global War on Terror. The estimation sub-sample still includes a 6-month period of Stop-Loss, an irregular component that had a significant effect on many of the population group forecasts. Despite the truncated time series span and the conspicuous effects of the irregular component, the study's forecasting models are generally effective. Incorporating larger observation sets after manpower levels stabilize in the future will likely yield more accurate forecasts.

While not the best technique evaluated, the Enlisted Staffing Goal Model's current forecast is surprisingly effective relative to the more sophisticated techniques developed in this thesis. The more distant into the future a forecast is made, the less accurate it generally is. Notably, the current model provides similar levels of effectiveness for both six- and twelve-month forecasts. Therefore, Headquarters Marine Corps could extend this method's prediction horizon from six to twelve months if required, with little bearing on the forecast's accuracy.

While the current method performs modestly better under the MAPE criterion than the other methods, the Holt-Winters exponential smoothing, multiplicative decomposition, and Box-Jenkins forecasting techniques outperform the status quo under the SSE and MAE goodness-of-fit measurements. The multiplicative decomposition method was superior to the other forecasting models using the SSE and MAE evaluation criteria. This thesis concludes that the multiplicative decomposition forecasting technique is the most effective univariate modeling technique to forecast future Marine Corps enlisted personnel inventory levels.

Of note is the six-month forecast returned by the decomposition method for the E1-E4 paygrade time series. For these paygrades and this specific forecast horizon, this technique surpasses the others in all evaluation criteria considered. This finding warrants further research of the technique's effectiveness on a larger pool of E1-E4 population group time series.

E. CHAPTER SUMMARY

This chapter evaluates Marine Corps enlisted personnel inventory forecasts developed by the Holt-Winters exponential smoothing, multiplicative decomposition, and Box-Jenkins techniques against the Enlisted Staffing Goal Model's current forecasting method. The predictions are evaluated against actual, out-of-sample inventory levels using several goodness-of-fit indicators including Mean Absolute Error Rate, Mean Absolute Error, and Sum of Squared Errors. Among the modeling techniques evaluated, the multiplicative decomposition performed the best overall and represents an improvement over the Marine Corps' current naïve forecasting method.

IX. CONCLUSIONS AND RECOMMENDATIONS

A. SUMMARY

Accurately forecasting future manpower inventory levels in a fundamental prerequisite for the development of an effective and functional staffing plan. The Enlisted Staffing Goal Model (ESGM) generates a proposed enlisted-force staffing plan that mitigates the dilemma of how to optimally meet service-manning requirements within the constraints of the Marine Corps' limited inventory of assignable personnel. In order for the staffing plan to be effective, the model's inputs must be accurate. One of these inputs, the forecast inventory available for assignment, is the weakest component of the ESGM process. Developing and incorporating a more accurate manpower inventory forecasting model will result in a closer to optimal distribution of personnel to meet the Marine Corps' staffing requirements and improve operational readiness.

A time series is a collection of observations in time. Although the values of individual observations cannot be predicted exactly, the distribution of stochastic time series observations commonly follows a discernable pattern. Statistical models can often describe these patterns. These models assume that the observations vary randomly about an underlying mean value that is a function of time. The time series may also be characterized by one or more behavioral components that may be isolated, modeled and incorporated into the forecast algorithm.

Univariate time series forecasting models make predictions by extrapolating the past behavior of a single variable of interest. Forecasting is appropriate for stochastic time series data when the underlying causes of variation do not change significantly over time. The forecasting process consists of five steps. This heuristic method involves formulating the problem, obtaining and preparing the data, selecting and applying forecasting methods, evaluating models, and using forecasts. Guided by this developmental framework, analysts can produce more accurate and efficient forecasts.

This thesis develops and evaluates manpower inventory forecasting models for 44 representative Marine Corps enlisted population groups using three univariate time series forecasting techniques. These methods are the Holt-Winters exponential smoothing,

multiplicative decomposition, and Box-Jenkins autoregressive integrated moving average forecasting models. Historical personnel strength levels obtained from the Marine Corps Total Force Data Warehouse comprise the observation sets employed in this study.

The Holt-Winters exponential smoothing method is a common univariate time series forecasting technique. This model smoothes irregular fluctuations by using weighted averages in an exponentially decreasing manner. The technique creates forecasts by estimating and extrapolating a linear trend while adjusting the data in each period by estimated seasonal indices.

Multiplicative decomposition is another effective method of forecasting univariate time series data. This technique is based on the concept that the underlying factors can be identified and isolated. These factors are trend, cycles, seasonality, and random variation. The analyst develops a model by first identifying and removing the component effects from the data. After these effects are isolated, the analyst creates a forecast by reassembling the modeled components.

The Box-Jenkins technique is a sophisticated approach by which to analyze time series data and extrapolate a forecast. This methodology provides a framework for preparing data, selecting a model, and creating forecasts and is commonly referred to as autoregressive integrated moving average (ARIMA) forecasting. This technique assumes any given time series observation can be modeled as a function of its past values and current and past values of random errors.

Models developed in this study using the Holt-Winters exponential smoothing, the multiplicative decomposition, and the Box-Jenkins forecasting techniques are compared to the current forecasting method and each other be applying a variety of statistical goodness-of-fit measurements. The evaluation techniques used are the Mean Absolute Percentage Error, the Sum of Squared Errors, and the Mean Absolute Error. Among the forecasting techniques evaluated, the multiplicative decomposition method performed best overall and represents an improvement over the Marine Corp's current forecasting method.

B. RECOMMENDATIONS

This thesis recommends Marine Corps Systems Command, Total Force Information Technology Systems develop and introduce a multiplicative decomposition forecasting model into the Enlisted Staffing Goal Model. This forecasting technique should be implemented in phases, starting with the E-1 through E-4 population groups.

C. SUGGESTED FURTHER STUDIES

Although there are no rigorous precepts for the number of time series observations required for effective forecast model parameter estimation, the literature generally suggest that the accuracy of predictions improve as the sample size increases. This thesis limits the estimation sub-sample to 36 observations to better reflect the impact of policy changes since the commencement of combat operations in Iraq and elsewhere in support of Global War on Terror. Incorporating larger sample sizes will likely yield more accurate forecasts. This thesis suggests future studies develop and evaluate models employing time series with 48 and 60 observations.

The scope of this study is limited to univariate time series forecasting techniques. Other approaches may be applicable to the manpower inventory level-forecasting problem. This thesis recommends future studies investigate the merit of techniques such as Markov chains and multivariate forecasting approaches.

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APPENDIX

This Appendix contains the actual six and twelve month inventory level and forecast results for each population group included in this study. Forecasts for the Holt-Winters exponential smoothing, multiplicative decomposition, Box-Jenkins, and the current method are provided in the tables below. The Absolute Percentage Error computation, described in Equation (5.25) is also provided. The tables also denote the specific ARIMA model specified for each time series during the Box-Jenkins model identification process.

Forecast Type	**Forecast Horizon**	**Actual Inventory**	**Predicted Inventory**	**Absolute Percentage Error**
Current Method	6-Months	130	121	6.9%
	12-Months	146	121	17.1%
Holt-Winters Exponential Smoothing	6-Months	130	167	28.5%
	12-Months	146	202	38.4%
Multiplicative Decomposition	6-Months	130	159	22.3%
	12-Months	146	168	15.1%
Box-Jenkins Box-Cox ARIMA (1,1,1)	6-Months	130	146	12.3%
	12-Months	146	159	8.9%

Table 21. 0211 E-5 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	117	121	3.4%
	12-Months	134	121	9.7%
Holt-Winters Exponential Smoothing	6-Months	117	137	17.1%
	12-Months	134	144	7.5%
Multiplicative Decomposition	6-Months	117	163	39.3%
	12-Months	134	163	21.6%
Box-Jenkins ARIMA (2,0,0)(1,0,0,)	6-Months	117	117	0%
	12-Months	134	111	17.9%

Table 22. 0211 E-6 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	86	62	27.9%
	12-Months	83	62	25.3%
Holt-Winters Exponential Smoothing	6-Months	86	66	23.3%
	12-Months	83	61	26.5%
Multiplicative Decomposition	6-Months	86	83	3.5%
	12-Months	83	81	2.4%
Box-Jenkins Logistic ARIMA(1,1,1,)	6-Months	86	71	17.4%
	12-Months	83	75	9.6%

Table 23. 0211 E-7 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	17	17	0%
	12-Months	16	17	6.3%
Holt-Winters Exponential Smoothing	6-Months	17	15	11.8%
	12-Months	16	12	25.0%
Multiplicative Decomposition	6-Months	17	23	35.3%
	12-Months	16	24	50.0%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	17	16	5.9%
	12-Months	16	15	6.3%

Table 24. 0211 E-8 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	9776	8571	12.3%
	12-Months	9662	8571	9.4%
Holt-Winters Exponential Smoothing	6-Months	9776	9125	6.7%
	12-Months	9662	9205	4.7%
Multiplicative Decomposition	6-Months	9776	9393	3.9%
	12-Months	9662	9872	2.2%
Box-Jenkins Logistic ARIMA (1,1,1)	6-Months	9776	9109	6.8%
	12-Months	9662	9215	4.6%

Table 25. 0311 E-1 – E-3 Forecast Results

Forecast Type	**Forecast Horizon**	**Actual Inventory**	**Predicted Inventory**	**Absolute Percentage Error**
Current Method	6-Months	3071	2855	7.0%
	12-Months	3140	2855	9.1%
Holt-Winters Exponential Smoothing	6-Months	3071	3028	1.4%
	12-Months	3140	3129	0.3%
Multiplicative Decomposition	6-Months	3071	2915	5.1%
	12-Months	3140	2847	9.3%
Box-Jenkins Box-Cox ARMA (1,1)	6-Months	3071	3044	0.9%
	12-Months	3140	3082	1.8%

Table 26. 0311 E-4 Forecast Results

Forecast Type	**Forecast Horizon**	**Actual Inventory**	**Predicted Inventory**	**Absolute Percentage Error**
Current Method	6-Months	1869	1757	6.0%
	12-Months	1880	1757	6.5%
Holt-Winters Exponential Smoothing	6-Months	1869	1762	5.7%
	12-Months	1880	1667	11.3%
Multiplicative Decomposition	6-Months	1869	1947	4.2%
	12-Months	1880	1932	2.8%
Box-Jenkins LOG ARIMA (1,1,1)	6-Months	1869	1830	2.1%
	12-Months	1880	1833	2.5%

Table 27. 0311 E-5 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	1387	1328	4.3%
	12-Months	1293	1328	2.7%
Holt-Winters Exponential Smoothing	6-Months	1387	1370	1.2%
	12-Months	1293	1354	4.7%
Multiplicative Decomposition	6-Months	1387	1425	2.7%
	12-Months	1293	1378	6.6%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	1387	1394	0.5%
	12-Months	1293	1401	7.7%

Table 28. 0369 E-6 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	711	697	2.0%
	12-Months	680	697	2.5%
Holt-Winters Exponential Smoothing	6-Months	711	716	0.07
	12-Months	680	722	6.2%
Multiplicative Decomposition	6-Months	711	667	6.2%
	12-Months	680	669	1.6%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	711	722	1.5%
	12-Months	680	723	5.9%

Table 29. 0369 E-7 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	178	191	7.3%
	12-Months	180	191	6.1%
Holt-Winters Exponential Smoothing	6-Months	178	208	16.9%
	12-Months	180	204	13.3%
Multiplicative Decomposition	6-Months	178	197	10.7%
	12-Months	180	189	5.0%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	178	216	21.3
	12-Months	180	217	20.6%

Table 30. 0369 E-8 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	64	66	3.1%
	12-Months	71	66	7.0%
Holt-Winters Exponential Smoothing	6-Months	64	67	4.7%
	12-Months	71	68	4.2%
Multiplicative Decomposition	6-Months	64	74	15.6%
	12-Months	71	75	5.6%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	64	73	14.1%
	12-Months	71	73	2.8%

Table 31. 0369 E-9 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	73	58	20.5%
	12-Months	88	58	34.1%
Holt-Winters Exponential Smoothing	6-Months	73	90	23.3%
	12-Months	88	103	17.0%
Multiplicative Decomposition	6-Months	73	76	4.1%
	12-Months	88	70	20.5%
Box-Jenkins ARIMA (2,1,0)(0,1,1)	6-Months	73	67	8.2%
	12-Months	88	82	6.8%

Table 32. 0321 E-1 – E-3 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	274	223	18.6%
	12-Months	305	223	26.9%
Holt-Winters Exponential Smoothing	6-Months	274	306	11.7%
	12-Months	305	347	13.8%
Multiplicative Decomposition	6-Months	274	237	13.5%
	12-Months	305	247	19.0%
Box-Jenkins Box-Cox (1,1,1)(1,1,1,)	6-Months	274	227	17.2%
	12-Months	305	225	26.2%

Table 33. 0321 E-4 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	248	252	1.6%
	12-Months	282	252	10.6%
Holt-Winters Exponential Smoothing	6-Months	248	256	3.2%
	12-Months	282	258	8.5%
Multiplicative Decomposition	6-Months	248	259	4.4%
	12-Months	282	262	7.6%
Box-Jenkins LOG ARIMA (2,0,0)(1,0,0)	6-Months	248	249	0.4%
	12-Months	282	248	12.1%

Table 34. 0321 E-5 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	141	129	8.5%
	12-Months	133	129	3.0%
Holt-Winters Exponential Smoothing	6-Months	141	130	7.8%
	12-Months	133	126	5.3%
Multiplicative Decomposition	6-Months	141	153	8.5%
	12-Months	133	151	13.5%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	141	132	6.4%
	12-Months	133	130	2.3%

Table 35. 0321 E-6 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	93	91	2.2%
	12-Months	87	91	4.4%
Holt-Winters Exponential Smoothing	6-Months	93	91	2.2%
	12-Months	87	90	3.4%
Multiplicative Decomposition	6-Months	93	90	3.3%
	12-Months	87	90	3.4%
Box-Jenkins ARIMA (2,0,0) (1,0,0)	6-Months	93	97	4.3%
	12-Months	87	98	12.6%

Table 36. 0321 E-7 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	23	26	13.0%
	12-Months	27	26	3.7%
Holt-Winters Exponential Smoothing	6-Months	23	25	8.7%
	12-Months	27	27	0%
Multiplicative Decomposition	6-Months	23	30	30.4%
	12-Months	27	31	14.8%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	23	26	13.0%
	12-Months	27	24	11.1%

Table 37. 0321 E-8 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	10	8	20%
	12-Months	11	8	27.3%
Holt-Winters Exponential Smoothing	6-Months	10	9	10%
	12-Months	11	9	18.2%
Multiplicative Decomposition	6-Months	10	10	0%
	12-Months	11	11	0%
Box-Jenkins	6-Months	10	9	10%
Box-Cox (1.5) ARIMA (1,1,1)	12-Months	11	9	18.2%

Table 38. 0321 E-9 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	83	85	2.4%
	12-Months	114	85	25.4%
Holt-Winters Exponential Smoothing	6-Months	83	109	31.3%
	12-Months	114	118	3.5%
Multiplicative Decomposition	6-Months	83	97	16.7%
	12-Months	114	104	8.8%
Box-Jenkins	6-Months	83	105	26.5%
Box-Cox (1.5) ARIMA (1,1,1)	12-Months	114	111	2.6%

Table 39. 2336 E-5 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	110	100	9.1%
	12-Months	101	100	1.0%
Holt-Winters Exponential Smoothing	6-Months	110	100	9.1%
	12-Months	101	102	1.0%
Multiplicative Decomposition	6-Months	110	100	9.1%
	12-Months	101	100	1.0%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	110	101	8.2%
	12-Months	101	102	1.0%

Table 40. 2336 E-6 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	71	57	19.7%
	12-Months	56	57	1.8%
Holt-Winters Exponential Smoothing	6-Months	71	45	36.6%
	12-Months	56	28	50.0%
Multiplicative Decomposition	6-Months	71	65	8.5%
	12-Months	56	64	14.3%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	71	66	7.0%
	12-Months	56	65	16.1%

Table 41. 2336 E-7 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	22	26	18.2%
	12-Months	23	26	13.0%
Holt-Winters Exponential Smoothing	6-Months	22	26	18.2%
	12-Months	23	24	4.3%
Multiplicative Decomposition	6-Months	22	30	36.4%
	12-Months	23	29	26.1%
Box-Jenkins	6-Months	22	28	27.3%
Box-Cox (1.5) ARMA (1,1)	12-Months	23	28	21.8%

Table 42. 2336 E-8 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	7	8	14.3%
	12-Months	9	8	11.1%
Holt-Winters Exponential Smoothing	6-Months	7	8	14.3%
	12-Months	9	8	11.1%
Multiplicative Decomposition	6-Months	7	9	28.6%
	12-Months	9	9	0%
Box-Jenkins	6-Months	7	8	14.3%
ARIMA (2,0,0)(1,0,0)	12-Months	9	8	11.1%

Table 43. 2336 E-9 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	1159	1211	4.5%
	12-Months	1147	1211	4.5%
Holt-Winters Exponential Smoothing	6-Months	1159	1301	12.3%
	12-Months	1147	1308	14.0%
Multiplicative Decomposition	6-Months	1159	1110	4.2%
	12-Months	1147	1076	6.2%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	1159	1290	11.3%
	12-Months	1147	1331	16.0%

Table 44. 5811 E-1 – E-3 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	618	556	10.0%
	12-Months	679	556	18.1%
Holt-Winters Exponential Smoothing	6-Months	618	525	15.0%
	12-Months	679	474	30.2%
Multiplicative Decomposition	6-Months	618	630	1.9%
	12-Months	679	611	10.0%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	618	573	7.3%
	12-Months	679	555	18.3%

Table 45. 5811 E-4 Forecast Results

Forecast Type	**Forecast Horizon**	**Actual Inventory**	**Predicted Inventory**	**Absolute Percentage Error**
Current Method	6-Months	489	494	1.0%
	12-Months	505	494	2.2%
Holt-Winters Exponential Smoothing	6-Months	489	478	2.2%
	12-Months	505	505	0%
Multiplicative Decomposition	6-Months	489	561	14.7%
	12-Months	505	570	12.9%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	489	496	1.4%
	12-Months	505	493	2.4%

Table 46. 5811 E-5 Forecast Results

Forecast Type	**Forecast Horizon**	**Actual Inventory**	**Predicted Inventory**	**Absolute Percentage Error**
Current Method	6-Months	300	303	1.0%
	12-Months	259	303	17.0%
Holt-Winters Exponential Smoothing	6-Months	300	292	2.7%
	12-Months	259	271	4.6%
Multiplicative Decomposition	6-Months	300	334	11.3%
	12-Months	259	332	28.2%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	300	321	7.0%
	12-Months	259	313	20.8%

Table 47. 5811 E-6 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	136	163	19.9%
	12-Months	154	163	5.8%
Holt-Winters Exponential Smoothing	6-Months	136	177	30.1%
	12-Months	154	193	25.3%
Multiplicative Decomposition	6-Months	136	158	16.2%
	12-Months	154	160	2.6%
Box-Jenkins	6-Months	136	159	16.9%
LOG ARIMA (2,0,0)(1,0,0)	12-Months	154	161	4.5%

Table 48. 5811 E-7 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	56	46	17.9%
	12-Months	50	46	32.0%
Holt-Winters Exponential Smoothing	6-Months	56	47	16.1%
	12-Months	50	43	14.0%
Multiplicative Decomposition	6-Months	56	57	1.8%
	12-Months	50	56	12.0%
Box-Jenkins	6-Months	56	55	1.8%
ARIMA (2,0,0)(1,0,0)	12-Months	50	57	14.0%

Table 49. 5811 E-8 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	20	18	10.0%
	12-Months	18	18	0%
Holt-Winters Exponential Smoothing	6-Months	20	18	10.0%
	12-Months	18	18	0%
Multiplicative Decomposition	6-Months	20	16	20.0%
	12-Months	18	15	16.7%
Box-Jenkins LOG ARIMA (1,1,1)	6-Months	20	17	15.0%
	12-Months	18	17	5.6%

Table 50. 5811 E-9 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	49	68	38.8%
	12-Months	86	68	20.9%
Holt-Winters Exponential Smoothing	6-Months	49	61	24.5%
	12-Months	86	40	53.5%
Multiplicative Decomposition	6-Months	49	32	34.7%
	12-Months	86	2	97.7%
Box-Jenkins LOGISTIC ARIMA (1,1,1)	6-Months	49	69	40.8%
	12-Months	86	57	33.7%

Table 51. 6092 E-1 – E-3 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	144	163	13.2%
	12-Months	150	163	8.7%
Holt-Winters Exponential Smoothing	6-Months	144	139	3.5%
	12-Months	150	124	17.3%
Multiplicative Decomposition	6-Months	144	169	17.4%
	12-Months	150	171	14.0%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	144	154	6.9%
	12-Months	150	157	4.7%

Table 52. 6092 E-4 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	124	115	7.3%
	12-Months	118	115	2.6%
Holt-Winters Exponential Smoothing	6-Months	124	119	4.0%
	12-Months	118	117	0.8%
Multiplicative Decomposition	6-Months	124	126	1.6%
	12-Months	118	125	5.9%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	124	120	3.2%
	12-Months	118	119	0.8%

Table 53. 6092 E-5 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	61	61	0%
	12-Months	56	61	8.9%
Holt-Winters Exponential Smoothing	6-Months	61	62	1.6%
	12-Months	56	62	10.7%
Multiplicative Decomposition	6-Months	61	62	1.6%
	12-Months	56	60	7.1%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	61	61	0%
	12-Months	56	61	8.9%

Table 54. 6092 E-6 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	27	25	7.4%
	12-Months	28	25	10.7%
Holt-Winters Exponential Smoothing	6-Months	27	24	11.1%
	12-Months	28	22	21.4%
Multiplicative Decomposition	6-Months	27	32	18.5%
	12-Months	28	32	14.2%
Box-Jenkins LOG ARIMA (2,0,0)(1,0,0)	6-Months	27	32	18.5%
	12-Months	28	32	14.2%

Table 55. 6092 E-7 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	200	162	19.0%
	12-Months	175	162	7.4%
Holt-Winters Exponential Smoothing	6-Months	200	161	19.5%
	12-Months	175	159	9.1%
Multiplicative Decomposition	6-Months	200	175	12.5%
	12-Months	175	175	0%
Box-Jenkins	6-Months	200	166	17.0%
LOG ARIMA (2,0,0)(1,0,0)	12-Months	175	165	5.7%

Table 56. 6019 E-8 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	79	81	2.5%
	12-Months	87	81	6.9%
Holt-Winters Exponential Smoothing	6-Months	79	85	7.6%
	12-Months	87	85	2.3%
Multiplicative Decomposition	6-Months	79	87	10.1%
	12-Months	87	88	1.1%
Box-Jenkins	6-Months	79	83	4.8%
ARIMA (2,0,0)(1,0,0)	12-Months	87	82	5.7%

Table 57. 6019 E-9 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	213	219	2.8%
	12-Months	189	219	15.9%
Holt-Winters Exponential Smoothing	6-Months	213	218	2.3%
	12-Months	189	225	19.0%
Multiplicative Decomposition	6-Months	213	236	10.8%
	12-Months	189	256	35.4%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	213	203	4.7%
	12-Months	189	191	1.1%

Table 58. 7257 E-1 – E-3 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	137	128	6.6%
	12-Months	141	128	9.2%
Holt-Winters Exponential Smoothing	6-Months	137	135	1.5%
	12-Months	141	139	1.4%
Multiplicative Decomposition	6-Months	137	139	1.5%
	12-Months	141	140	0.7%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	137	129	5.8%
	12-Months	141	127	9.9%

Table 59. 7257 E-4 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	113	102	9.7%
	12-Months	112	102	8.9%
Holt-Winters Exponential Smoothing	6-Months	113	100	11.5%
	12-Months	112	103	8.0%
Multiplicative Decomposition	6-Months	113	106	6.2%
	12-Months	112	107	4.5%
Box-Jenkins Box-Cox (1.5) ARMA (1,1)	6-Months	113	112	0.9%
	12-Months	112	112	0%

Table 60. 7257 E-5 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	86	97	12.8%
	12-Months	81	97	18.6%
Holt-Winters Exponential Smoothing	6-Months	86	89	3.5%
	12-Months	81	77	4.9%
Multiplicative Decomposition	6-Months	86	99	15.1%
	12-Months	81	97	19.8%
Box-Jenkins LOG ARIMA (2,0,0) (1,0,0)	6-Months	86	106	23.3%
	12-Months	81	105	29.6%

Table 61. 7257 E-6 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	65	54	16.9%
	12-Months	60	54	10.0%
Holt-Winters Exponential Smoothing	6-Months	65	65	0%
	12-Months	60	70	14.3%
Multiplicative Decomposition	6-Months	65	53	18.5%
	12-Months	60	56	6.7%
Box-Jenkins LOG ARIMA (2,0,0)(1,0,0)	6-Months	65	52	20.0%
	12-Months	60	53	13.2%

Table 62. 7257 E-7 Forecast Results

Forecast Type	Forecast Horizon	Actual Inventory	Predicted Inventory	Absolute Percentage Error
Current Method	6-Months	19	23	21.1%
	12-Months	19	23	21.1%
Holt-Winters Exponential Smoothing	6-Months	19	21	10.5%
	12-Months	19	20	5.3%
Multiplicative Decomposition	6-Months	19	23	17.4%
	12-Months	19	23	7.4%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	19	22	15.8%
	12-Months	19	22	15.8%

Table 63. 7291 E-8 Forecast Results

Forecast Type	**Forecast Horizon**	**Actual Inventory**	**Predicted Inventory**	**Absolute Percentage Error**
Current Method	6-Months	4	4	0%
	12-Months	5	4	20.0%
Holt-Winters Exponential Smoothing	6-Months	4	4	0%
	12-Months	5	4	20.0%
Multiplicative Decomposition	6-Months	4	4	0%
	12-Months	5	4	20.0%
Box-Jenkins ARIMA (2,0,0)(1,0,0)	6-Months	4	4	0%
	12-Months	5	4	20.0%

Table 64. 7291 E-9 Forecast Results

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